Foundational Numeracy

MATH 1525

Introduction to Multiplication

Suppose that we wish to count the number of Oiler emblems pictured to the right. The emblems are arranged in **5** rows, and each row contains **6** emblems

Adding **5** separate groups of **6** emblems will give the total number of Oiler emblems.

- We can write this as

$$\underbrace{6 + 6 + 6 + 6 + 6 = 30}$$

The above is known as **repeated addition**, where each addend (in this case, **6**) is the exact same.

Multiplication is the process of repeated addition, but with different notation 😎

Multiplication is Repeated Addition

$$6 + 6 + 6 + 6 + 6 = 30$$

→

Repeated Addition

- \# of addends: **5**
- Value of each addend: **6**

In multiplication terms,

$$5 \times 6 = 30$$

Factor 1
(# of addends)

Factor 2
(value of each addend)

Product
(the result of any multiplication operation is always called the product)

Multiplication in Many Forms…

But they all mean the same thing!

$$5 \times 6 = 30$$

Read as "five times six equals thirty"

$5 \times 6 = 30$

$5 \cdot 6 = 30$

$(5)(6) = 30$

$(5)6 = 30$

$5(6) = 30$

Properties of Multiplication

Property of 0	The product of 0 and any other value is always 0 $0 \times 5 = \mathbf{0}$ $0 \ \times 1{,}000{,}000 = \mathbf{0}$
Property of 1	The product of 1 any other value is always that same value $1 \times \mathbf{9} = \mathbf{9}$ $1 \times \mathbf{6,342} = \mathbf{6,342}$
Commutative Property	Changing the order of factors does not change the product $2 \times 9 = \mathbf{18}$ $9 \times 2 = \mathbf{18}$
Associative Property	Changing the grouping of factors does not change the product $(2 \cdot 3) \cdot 4$ → $6 \cdot 4 = \mathbf{24}$ $2 \cdot (3 \cdot 4)$ → $2 \cdot 12 = \mathbf{24}$
Distributive Property	Multiplication distributes over addition $2(3+4)$ $(2 \cdot 3) + (2 \cdot 4)$ $6 + 8$ =14

Developing Number Sense: Factoring

When you are able to factor a number, you are able to see the parts and qualities that make up that same number at a deeper level, as if that number were under a microscope.

Through this understanding and the tool of mathematics, we can then perform whatever operations are required on that number so that we achieve a desired, but most importantly, a **useful** effect.

Factoring

- Factoring provides a deeper, practical understanding of prime numbers
  - Memorization of multiplication tables is not nearly as useful as the ability to factor / see individual parts of an expression

- Factoring gives way to being able to eventually find common multiples, common denominators (fractions), simplifying expressions, and other higher level practicalities of mathematics

Find the Factors of: 45

We know that:

Method 1: List

$$1 \times 45 = 45$$

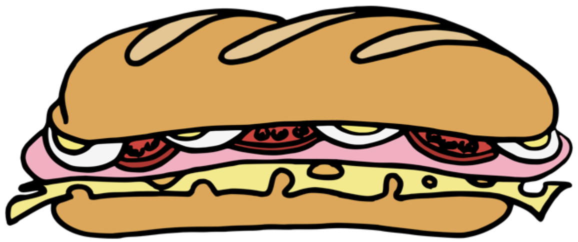
$$3 \times 15 = 45$$

$$5 \times 9 = 45$$

∴ The factors of 45 are: 1 , 3 , 5 , 9 , 15 , & 45

Find the Factors of: 45

Method 2: Sandwich



The “sandwich method” is a simple method of finding the factors of a number.

1 and 45 are the most obvious factors of 45, so start with those as your end pieces.

Now, work your way inwards until there are no other whole numbers that can work as factors of 45:

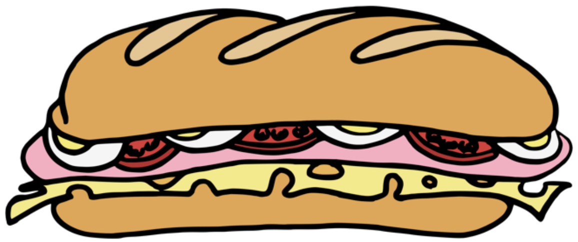
1					**45**	$(1 \times 45 = 45)$
1	**3**			**15**	45	$(3 \times 15 = 45)$
1	3	**5**	**9**	15	45	$(5 \times 9 = 45)$

Since we have worked our way inwards as much as possible, there are no more factors of 45. We are finished.

∴ The factors of 45 are: 1 , 3 , 5 , 9 , 15 , & 45

Find the Factors of: 28

Method 2: Sandwich



1 **28** $(1 \times 28 = 28)$

1 **2** **14** 28 $(2 \times 14 = 28)$

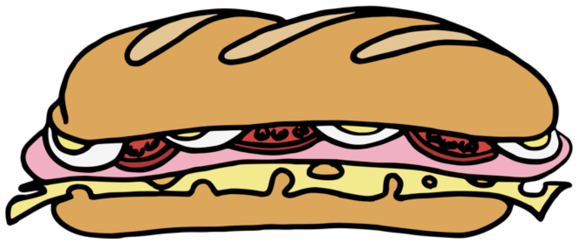
1 3 **4** **7** 15 45 $(4 \times 7 = 28)$

Since we have worked our way inwards as much as possible, there are no more factors of 28. We are finished.

∴ The factors of 28 are: 1 , 2 , 4 , 7 , 14 , & 28

Find the Factors of: 100

Method 2: Sandwich



1								100	$(1 \times 100 = 100)$
1	2						50	100	$(2 \times 50 = 100)$
1	2	4				25	50	100	$(4 \times 25 = 100)$
1	2	4	5		20	25	50	100	$(5 \times 20 = 100)$
1	2	4	5	10	20	25	50	100	$(10 \times 10 = 100)$

Since we have worked our way inwards as much as possible, there are no more factors of 100. We are finished.

$\therefore$ The factors of 100 are: 1 , 2 , 4 , 5 , 10 , 20 , 25 , 50 , 100

Exponents: Repeated Multiplication

Exponents, also called **powers**, are used when you multiply a number by itself. For example, if you were to multiply the number 2 by itself, you can show this in the multiplication form:

2×2 $(2)(2)$ $2 \cdot 2$

In **exponential notation**, this is written as :

$$2^2$$

"*two to the power of two*" or "*two <u>squared</u>*"

$$2^2 = 4$$

Exponents

If you were to multiply 2 by itself 3 times, you can show this in multiplication form:

$2 \times 2 \times 2$ $(2)(2)(2)$ $2 \cdot 2 \cdot 2$

In **exponential notation**, this is written as :

$$2^3$$

"*two to the power of three*" or "*two cubed*"

$$2^3 = 8$$

Base & Exponent

Exponent

$$2^3$$

Base

This DOES NOT MEAN

2×3

$= 6$

This DOES mean:

$2 \times 2 \times 2$

$= 8$

Base 2 Exponential Form: Examples

Exponential Notation	Multiplication Form	Numerical Form (Value)	Name, in Words
2^0	N/A	1	"Two to the power of zero"
2^1	2	2	"Two to the power of one"
2^2	2×2	4	"Two to the power of two" **"Two Squared"**
2^3	$2 \times 2 \times 2$	8	"Two to the power of three" **"Two Cubed"**
2^4	$2 \times 2 \times 2 \times 2$	16	"Two to the power of four"
2^5	$2 \times 2 \times 2 \times 2 \times 2$	32	"Two to the power of five"

Base 3 Exponential Form: Examples

Exponential Notation	Multiplication Form	Numerical Form (Value)	Name, in Words
3^0	N/A	1	"Three to the power of zero"
3^1	3	3	"Three to the power of one"
3^2	3×3	9	"Three to the power of two" **"Three Squared"**
3^3	$3 \times 3 \times 3$	27	"Three to the power of three" **"Three Cubed"**
3^4	$3 \times 3 \times 3 \times 3$	81	"Three to the power of four"
3^5	$3 \times 3 \times 3 \times 3 \times 3$	243	"Three to the power of five"

If the exponents are 1 or 0…

$$x^0 = 1$$

In the above example, "x" is used to represent ANY number

Any number that has an exponent of 0 will always equal to 1

$$5^0 = 1$$

$$100^0 = 1$$

$$1{,}348^0 = 1$$

$$x^1 = x$$

In the above example, "x" is used to represent ANY number

Any number that has an exponent of 1 will always equals to that original number

$$5^1 = 5$$

$$100^1 = 100$$

$$1{,}348^1 = 1{,}348$$

Square Roots

A **square root** of a number is one of two identical factors of a number. If the square roots of a number are whole numbers, this is called a **perfect square**.

To find out the **base** of a **squared** value, we need to take a **square root** of that number. For example, we know that **two squared**, or $\mathbf{2^2}$, is **4**. This means that *the **square root** of **4** is **2***. We also know that **3 squared**, or $\mathbf{3^2}$, is **9**. This means that *the **square root** of **9** is **3***.

Square Root Form	Numerical Value
$\sqrt{4} =$	2
$\sqrt{9} =$	3
$\sqrt{16} =$	4
$\sqrt{25} =$	5
$\sqrt{36} =$	6
$\sqrt{49} =$	7
$\sqrt{64} =$	8
$\sqrt{81} =$	9
$\sqrt{100} =$	10
$\sqrt{121} =$	11
$\sqrt{144} =$	12
$\sqrt{169} =$	13
$\sqrt{196} =$	14
$\sqrt{225} =$	15

because→

Exponent Notation	Numerical Value
$2^2 =$	4
$3^2 =$	9
$4^2 =$	16
$5^2 =$	25
$6^2 =$	36
$7^2 =$	49
$8^2 =$	64
$9^2 =$	81
$10^2 =$	100
$11^2 =$	121
$12^2 =$	144
$13^2 =$	169
$14^2 =$	196
$15^2 =$	225

Squares & Area

One of the most common applications of multiplication in the real world is through the concept of **Area:** the size of a surface in square units

(can use m, cm, mm, feet)

To calculate the area of an object, multiply its length by its base ($length \times height$)

*notice how this can be applied to square roots

Class Discussion: Are there any other ways To show a garden that would also have an area of $16\ ft^2$?

If each square has a side length of exactly 1 ft, then:

4 ft$\times$ 4ft$=$ $16\ ft^2$

Comparing Factors…

Consider the following numbers, listed with its factors:

12 → The factors of 12 are: $1, 2, 3, 4, 6,$ and 12

15 → The factors of 15 are: $1, 3, 5,$ and 15

7 → The factors of 7 are: 1 and 7

<u>Class Discussion</u>

Do you notice any similarities between these? Differences?

The biggest similarity between the three numbers is that they all have (1) and (itself) as factors. The biggest difference, however, is that 12 and 15 have more than just (1) and (itself) as factors. The reason for this is that 12 and 15 are both ***composite*** *numbers, whereas 7 is known as a* ***prime*** *number.*

Prime or Composite?

Composite Numbers

- are numbers that have more than just (1) and (itself) as factors. Naturally, all numbers that are not prime, are considered to be composite numbers. Examples: 4, 6, 8, 9, 12, 15, 36, 100

Prime Numbers

- are numbers that have exactly two different factors, being (1) and (itself).
- Examples: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37. *All of these numbers have only two factors: 1 and itself.*

Class Discussion:

Who remembers seeing these
in grade school?

What were your experiences with these?

***Memorization of multiplication tables is not nearly as useful as the ability to factor/break down and see individual parts of an expression

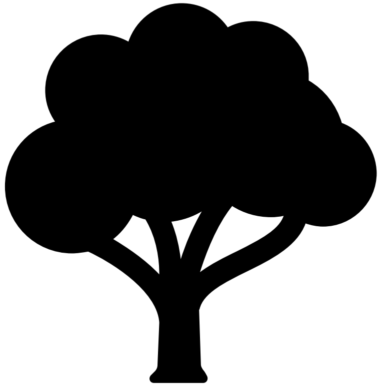
Multiplication Table - 20x20

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	**1**	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	2	**4**	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
3	3	6	**9**	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60
4	4	8	12	**16**	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80
5	5	10	15	20	**25**	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
6	6	12	18	24	30	**36**	42	48	54	60	66	72	78	84	90	96	102	108	114	120
7	7	14	21	28	35	42	**49**	56	63	70	77	84	91	98	105	112	119	126	133	140
8	8	16	24	32	40	48	56	**64**	72	80	88	96	104	112	120	128	136	144	152	160
9	9	18	27	36	45	54	63	72	**81**	90	99	108	117	126	135	144	153	162	171	180
10	10	20	30	40	50	60	70	80	90	**100**	110	120	130	140	150	160	170	180	190	200
11	11	22	33	44	55	66	77	88	99	110	**121**	132	143	154	165	176	187	198	209	220
12	12	24	36	48	60	72	84	96	108	120	132	**144**	156	168	180	192	204	216	228	240
13	13	26	39	52	65	78	91	104	117	130	143	156	**169**	182	195	208	221	234	247	260
14	14	28	42	56	70	84	98	112	126	140	154	168	182	**196**	210	224	238	252	266	280
15	15	30	45	60	75	90	105	120	135	150	165	180	195	210	**225**	240	255	270	285	300
16	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	**256**	272	288	304	320
17	17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	**289**	306	323	340
18	18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	**324**	342	360
19	19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	**361**	380
20	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	**400**

Prime Factorization

The prime factorization of a number is the factorization (breaking down) of a number to the point that all of its factors are simplified into prime numbers;

It is the process of breaking down a number (into its factors) until you can no longer break it down because it is prime

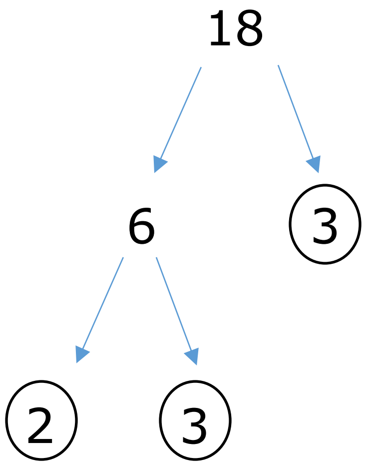


To find the prime factorization of a number, we will use **factor trees**

The key to factor trees is breaking down numbers into factors in any way possible. It doesn’t even matter how you start, just start factoring! Once one of the factors are a *prime number*, **circle** it to indicate you are finished with that branch, as you cannot continue to break down that prime number. Continue breaking down the composite factors into prime factors until there are no more numbers to break down.

Prime Numbers to look out for: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37

Find the Prime Factorization of: 18

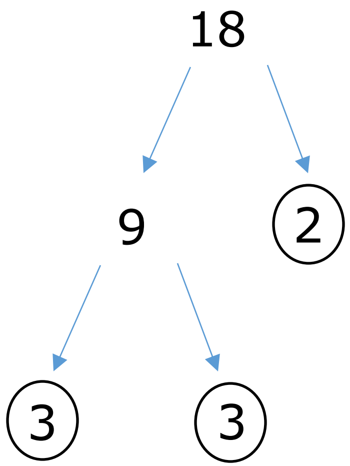


The prime factorization of 18 is:

$2 \cdot 3 \cdot 3$

or

$2 \cdot 3^2$



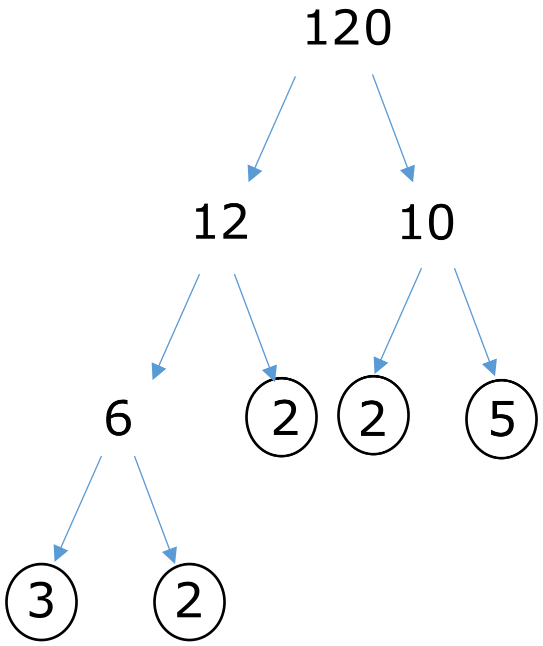
The prime factorization of 18 is:

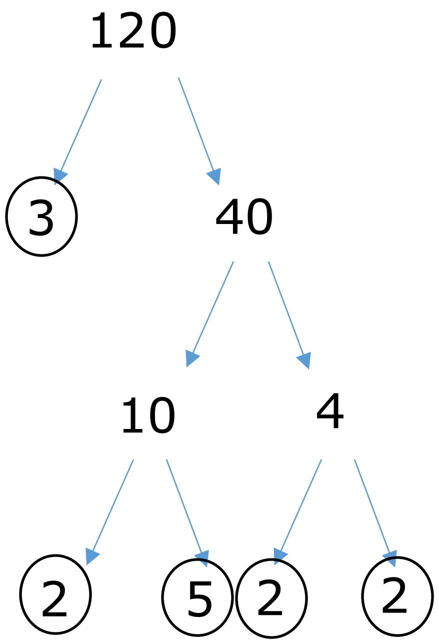
$2 \cdot 3 \cdot 3$

or

$2 \cdot 3^2$

Find the Prime Factorization of: 120





The prime factorization of 120 is:

$2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$

or

$2^3 \cdot 3 \cdot 5$

Class Discussion: What other factors could we have used to **start** our factor trees?

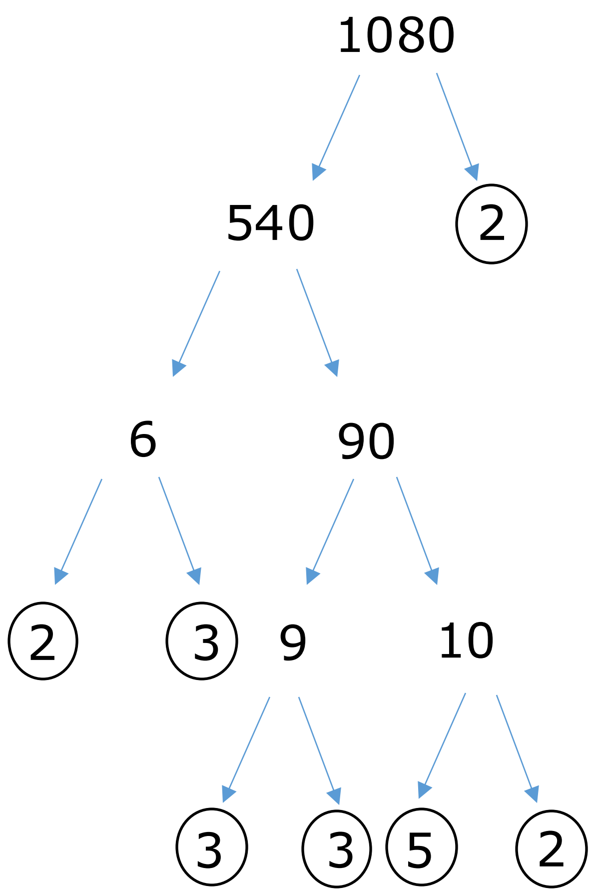
The prime factorization of 120 is:

$2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$

or

$2^3 \cdot 3 \cdot 5$

Find the Prime Factorization of: 1080



The prime factorization of 1080 is:

$$2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 \cdot 5$$

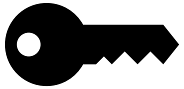
or

$$2^3 \cdot 3^3 \cdot 5$$

It doesn't matter how you start a factor tree…

Just keep breaking down the composite numbers until they are **prime**!

Long Multiplication: Taking it Old School

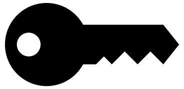
The  to long multiplication: **place values**

$$\begin{array}{r} 24 \\ \times \quad 8 \\ \hline \end{array}$$

Step 1:

Multiply the ones
(this will *always* be to the most right in any long multiplication question)

Long Multiplication: Taking it Old School

The  to long multiplication: **place values**

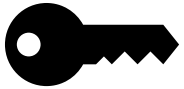
$$\begin{array}{r} {}^{3} \\ 24 \\ \times \quad 8 \\ \hline 2 \end{array}$$

$8 \times 4 = 32$

Step 2:

Since $8 \times 4 = 32$, 32 means 3 tens and 2 ones. The 2 ones can stay in the ones spot, and we will carry the 3 tens into the tens place value

Long Multiplication: Taking it Old School

The  to long multiplication: **place values**

```
   3
  24
×  8
____
   2
```

Step 3:

Multiply the 8 to the number in the tens place value.

In this case, we will multiply the 8 by the 2 (in 24)

Long Multiplication: Taking it Old School

The 🔑 to long multiplication: **place values**

$$\begin{array}{r} {}^{3} \\ 24 \\ \times 8 \\ \hline 2 \end{array}$$

$8 \times 2 = 16$

Step 4:

Add the **carry** to your latest product

$16 + \mathbf{3} = 19$

Long Multiplication: Taking it Old School

The to long multiplication: **place values**

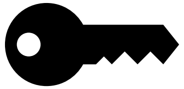
$$\begin{array}{r} {}^{3} \\ 24 \\ \times \quad 8 \\ \hline 192 \end{array}$$

$16 + 3 = 19$

Step 5:

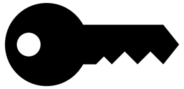
- Write a 9 in the tens spot
- Write a 1 in a newly created hundreds spot

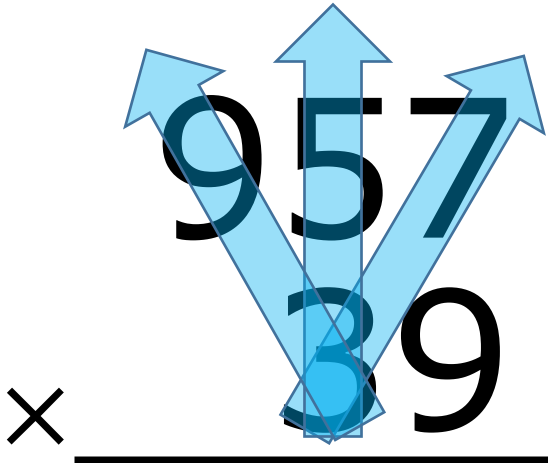
Long Multiplication: Taking it Old School

The  to long multiplication: **place values**

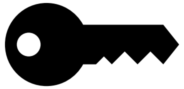
$$\begin{array}{r} 957 \\ \times \quad 39 \\ \hline \end{array}$$

Long Multiplication: Taking it Old School

The  to long multiplication: **place values**



Long Multiplication: Taking it Old School

The  to long multiplication: **place values**

$$\begin{array}{r} 957 \\ \times \quad 39 \\ \hline \end{array}$$

Long Multiplication: Taking it Old School

The to long multiplication: **place values**

$$\begin{array}{r} {\scriptstyle 6} \\ 957 \\ \times \quad 39 \\ \hline 3 \end{array}$$

Step 1:

Multiply the numbers in the Ones place value on the very right

$9 \times 7 = 63$

63 means 6 (10s) and 3 (1s), so the 3 can stay in the ones spot, and we carry the 6 tens to the next place value over (the tens place value)

Long Multiplication: Taking it Old School

The 🔑 to long multiplication: **place values**

$$\begin{array}{r} {}^{5\;\,6} \\ 957 \\ \times \quad 39 \\ \hline 13 \end{array}$$

Step 2:

Multiply your current anchor with the next number

$9 \times 5 = 45$

We have a **carry** of **6**, so we add that to our latest product

$45 + \mathbf{6} = 51$

Write a 1 in the tens spot, and carry the 5 to the next place value over (hundreds)

Long Multiplication: Taking it Old School

The 🔑 to long multiplication: **place values**

$$\begin{array}{r} {}^{5\ 6}\\ 957 \\ \times \quad 39 \\ \hline 8613 \end{array}$$

Step 3:

Multiply your current anchor with the next number

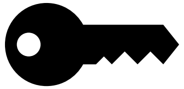
$9 \times 9 = 81$

We have a **carry** of **5**, so we add that to our latest product

$81 + \mathbf{5} = 86$

Write a 6 in the hundreds spot
Write an 8 in a newly created thousands spot

Long Multiplication: Taking it Old School

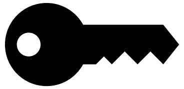
The  to long multiplication: **place values**

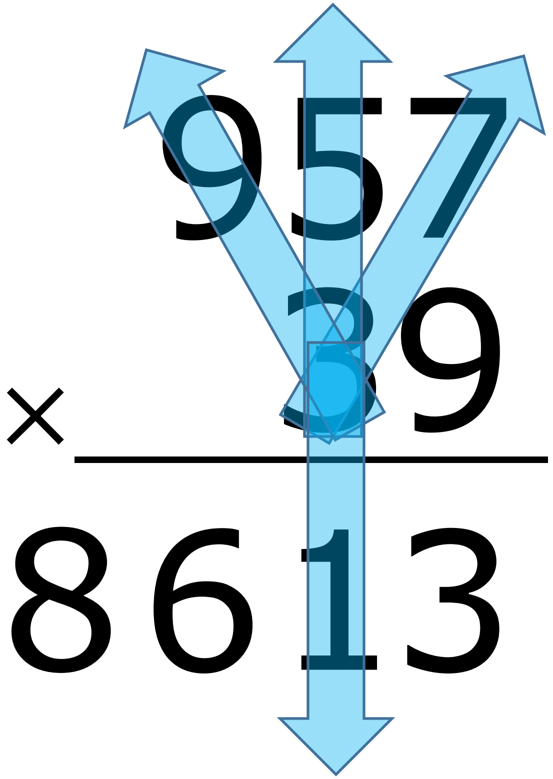
$$
\begin{array}{r}
{\scriptstyle 5\ \ 6\ \ \ } \\
957 \\
\times \quad 39 \\
\hline
8613
\end{array}
$$

Step 4:

Before we start multiplying our next anchor, erase all your previously carried values as those only apply to the first anchor

Long Multiplication: Taking it Old School

The  to long multiplication: **place values**



Step 5:

Begin the multiplication process with the next anchor.

Because we are multiplying a new anchor, the values we are about to calculate will start under this very same anchor

Long Multiplication: Taking it Old School

The [key] to long multiplication: **place values**

```
   2
 957
×  39
―――――
8613
   1
```

Step 6:

Multiply your new anchor (3) with the number in the ones place (7)

$3 \times 7 = 21$

Again, because we are multiplying a new anchor, the values we are about to calculate will start under this very same anchor

So, we will write a 1 under the current anchor, and carry the 2 to the next place value over

Long Multiplication: Taking it Old School

The 🔑 to long multiplication: **place values**

$$\begin{array}{r} {}^{1\ 2}\\ 957 \\ \times\quad 39 \\ \hline 8613 \\ 71 \end{array}$$

Step 7:

Multiply your new anchor (3) with the number in the tens place (5)

$3 \times 5 = 15$

We have a **carry** of **2**, so we add that to our latest product

$15 + 2 = 17$

Continuing on the *same* line, write a 7 and carry the 1 to the next place value over

Long Multiplication: Taking it Old School

The key to long multiplication: **place values**

```
   1 2
   957
×   39
──────
  8613
 2871
```

Step 8:

Multiply your new anchor (3) with the number in the hundreds place (9)

$3 \times 9 = 27$

We have a **carry** of **1**, so we add that to our latest product

$27 + \mathbf{1} = 28$

Continuing on the *same* line, write an 8 in the thousands place value, and write a 2 in a newly created ten thousands place value

Long Multiplication: Taking it Old School

$$\begin{array}{r} {}^{1\ 2} \\ 957 \\ \times \quad 39 \\ \hline {}^{1\ 1}\ 8613 \\ +\,28710 \\ \hline 37323 \end{array}$$

Step 9:

You may now erase your carried values

Add your two numbers at the bottom to give a solution (You might have to carry!)

"Bring down" any numbers that don't have an adding "partner"