Foundational Numeracy

MATH 1525

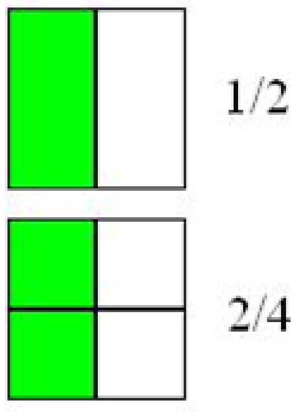
Working with Fractions

What are Fractions?

Whole numbers are used to count whole items

For example, you can count:

- How many houses there are in a neighborhood
- How many pets a family owns
- How many people got vaccinated



Fractions are used to refer to **parts** of an entire whole

For example:

- Of all the houses in the neighborhood, how many of them are **blue**?
- Of all their pets, how many of them are dogs?
- Of all people vaccinated, how many received the Pfizer-BioNTech?

Dictionary

Search for a word

frac·tion

/ˈfrakSH(ə)n/

See definitions in:

All | Mathematics | Chemistry | Ecclesiastical

noun

1. a numerical quantity that is not a whole number (e.g. 1/2, 0.5).
2. a small or tiny part, amount, or proportion of something.
   "he hesitated for **a fraction of** a second"
   Similar: tiny part, small part, fragment, snippet, snatch, smattering

Fractions represent a part of (1) entire whole

Division Properties of 1 & 0

Why is something considered whole?

$$\frac{6}{6} = 1$$

WHY?

$6 \div 6 = 1$

Because

$1 \times 6 = 6$

Division Properties of 1 & 0

Why is something considered whole?

$$\frac{54}{1} = 54$$

WHY?

$$54 \div 1 = 54$$

Because

$$54 \times 1 = 54$$

Division Properties of 1 & 0

Why is something considered whole?

$$\frac{0}{6} = 0$$

WHY?

$$0 \div 6 = 0$$

Because

$$0 \times 6 = 0$$

Division Properties of 1 & 0

Why is something considered whole?

$$\frac{6}{0} = \text{undefined}$$

WHY?

$$6 \div 0 = \text{undefined}$$

Because

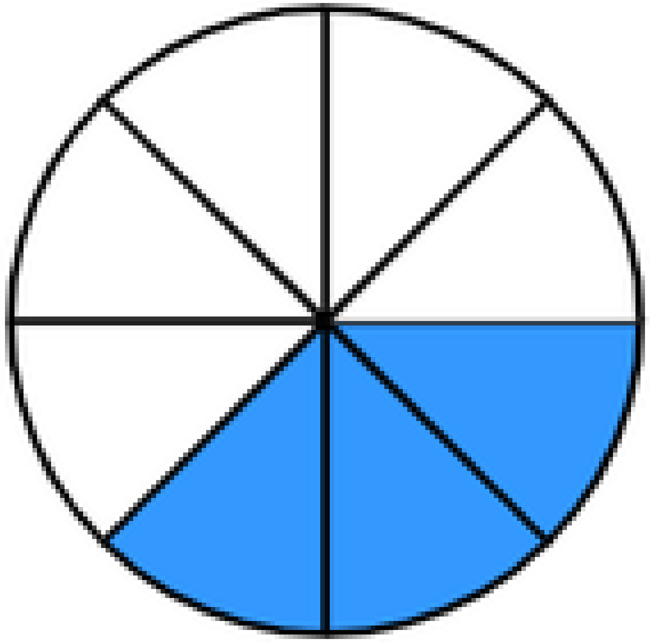
No matter what, anything you multiply by 0 will never be 6. It will always be 0

Parts of a Fraction

Names	Fraction	Meaning
Numerator	3	Number of parts being considered
Denominator	4	Number of equal parts in the entire whole

Fractions

Write a Fraction to represent the shaded areas of the following figures:



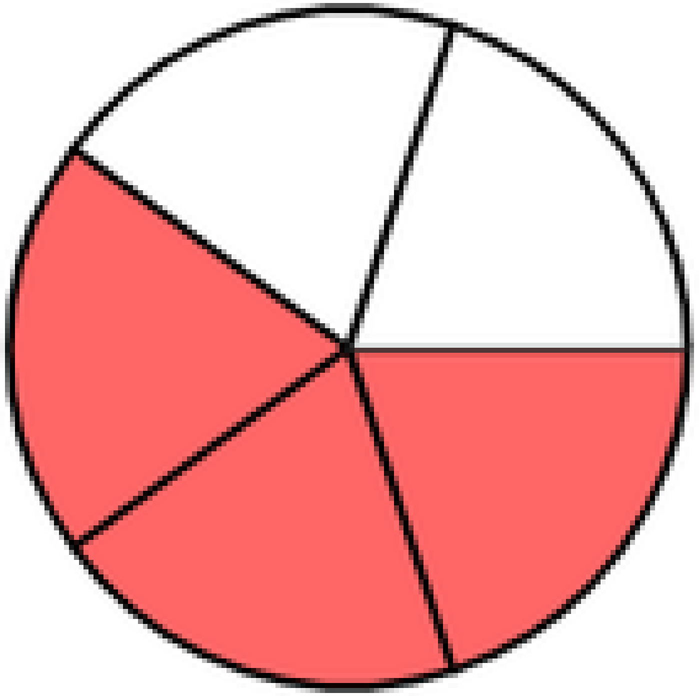
$$\frac{3}{8}$$

3 (Number of Parts Shaded)

8 (Number of Equal Parts)

Fractions

Write a Fraction to represent the shaded areas of the following figures:



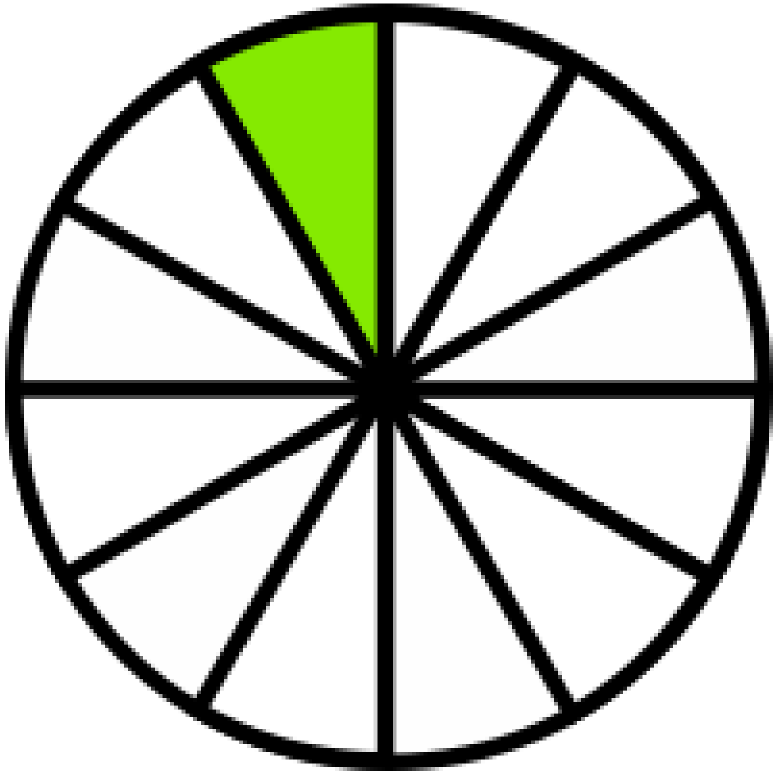
$$\frac{3}{5}$$

(Number of Parts Shaded)

(Number of Equal Parts)

Fractions

Write a Fraction to represent the shaded areas of the following figures:



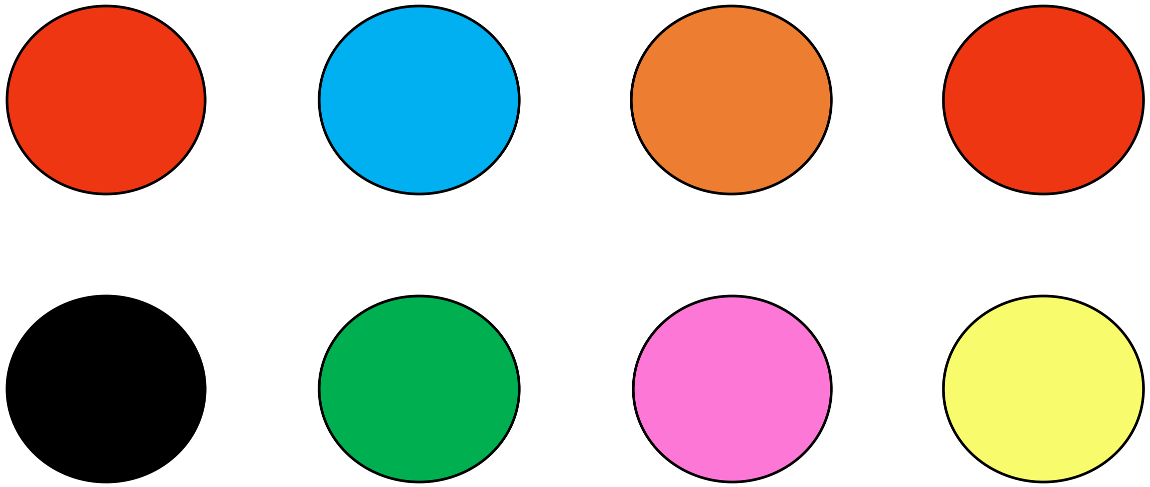
$$\frac{1}{12}$$

(Number of Parts Shaded)

(Number of Equal Parts)

Fractions

Write a Fraction to represent the number of red circles

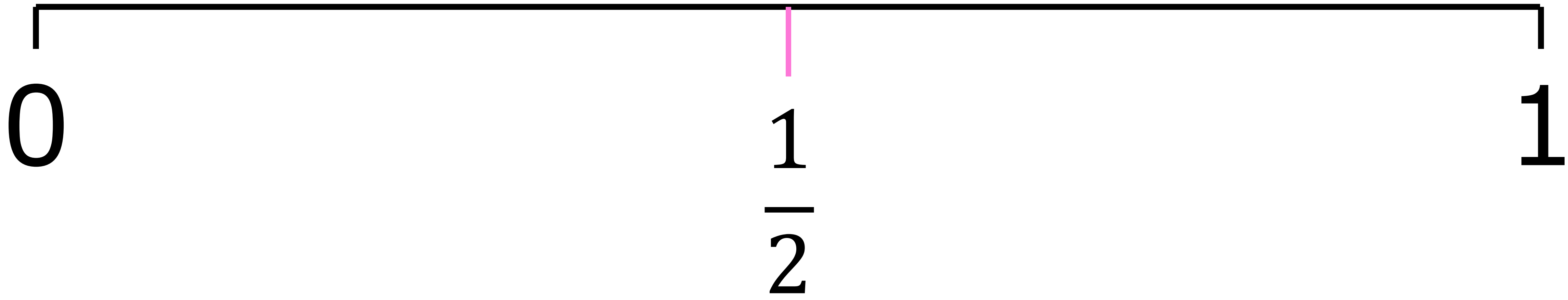


$$\frac{2}{8}$$

2 (Number of **RED** circles)

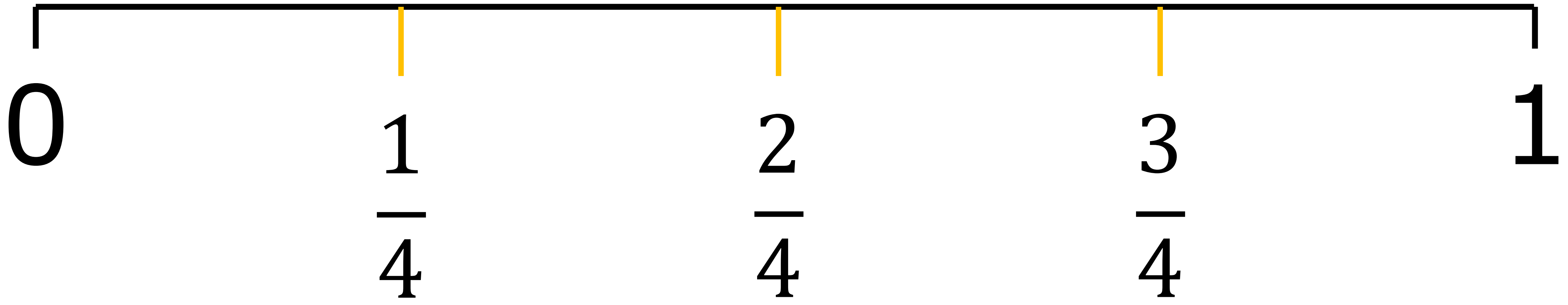
8 (Number of Circles)

Fractions on a Number Line

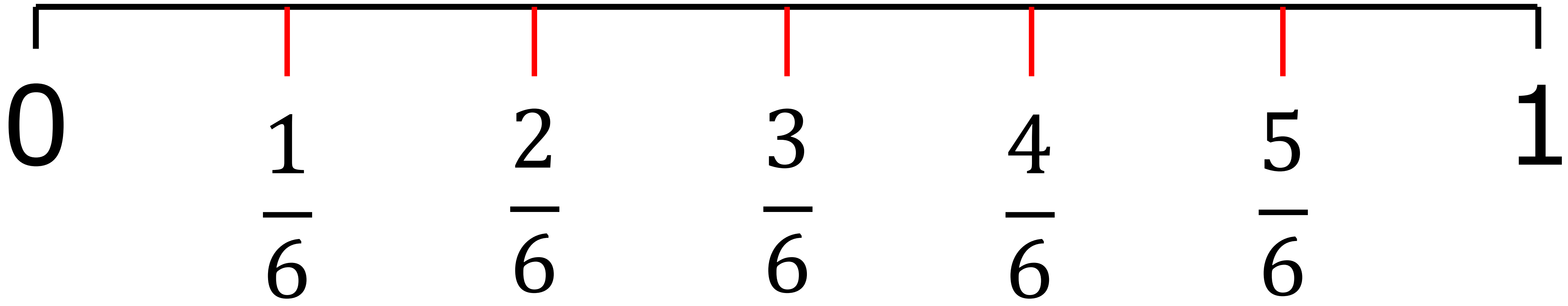


Fractions on a Number Line

Fractions on a Number Line



Fractions on a Number Line



Proper Fractions

A **proper fraction** occurs when the numerator is less than its denominator. Proper fractions are used to represent parts that do not form (1) whole

$$\frac{1}{4} \qquad \frac{1}{8} \qquad \frac{176}{5000} \qquad \frac{17}{34}$$

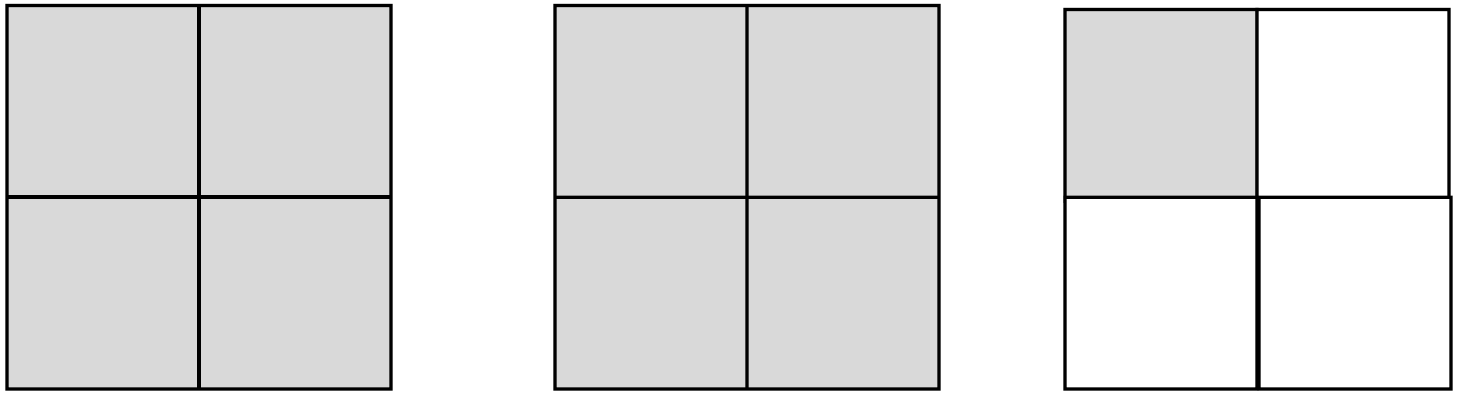
Improper Fractions & Mixed Numbers

An **improper fraction** occurs when the numerator is greater than, or equal, to its denominator. Improper fractions are always either equal to 1 whole, or greater than 1 whole

$$\frac{28}{28} \qquad \frac{4}{3} \qquad \frac{100}{6} \qquad \frac{35}{35}$$

Improper Fractions & Mixed Numbers

Mixed Numbers contains a whole number, alongside a fraction. It indicates that you Have more than (1) whole



1 whole | 1 whole | $\frac{1}{4}$ of 1 whole

$$2\frac{1}{4}$$

This DOES NOT represent $\frac{9}{12}$ … because you already have 2 wholes!

Improper Fractions & Mixed Numbers

Mixed Numbers and **Improper Fractions** are related because they tell the same thing:
You have *more* than 1 whole

$$3\frac{1}{2} \qquad \frac{7}{2}$$

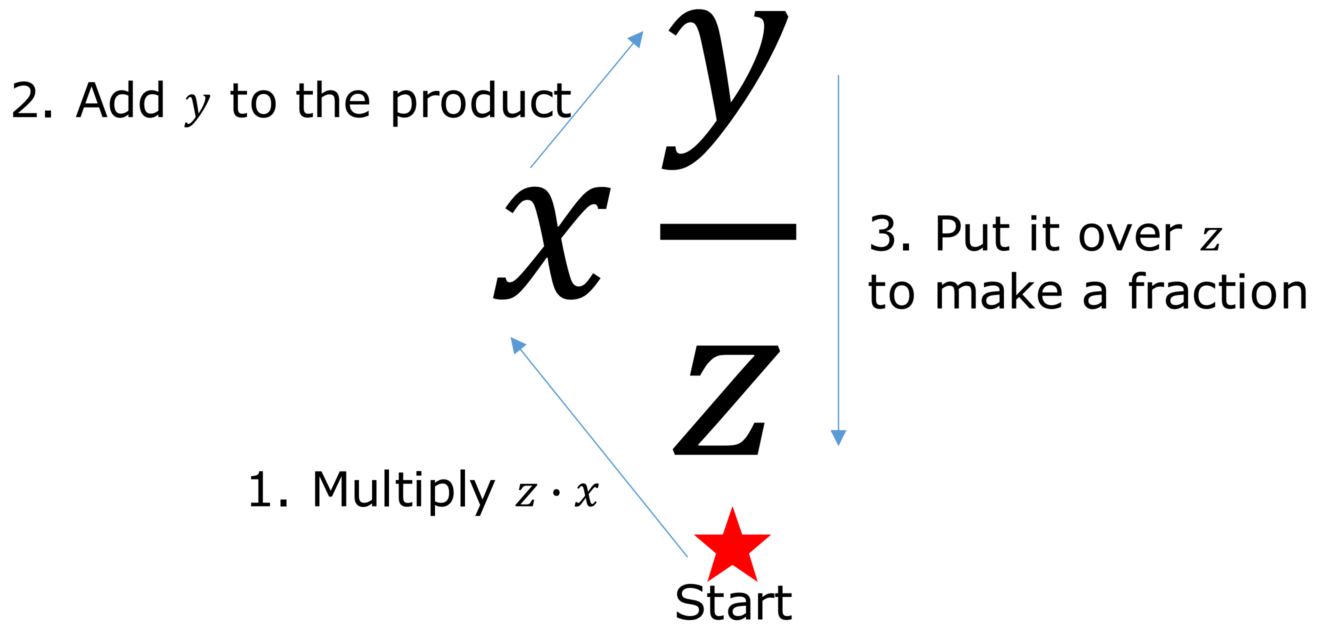
$$10\frac{3}{4} \qquad \frac{43}{4}$$

Converting Mixed Numbers to Improper Fractions

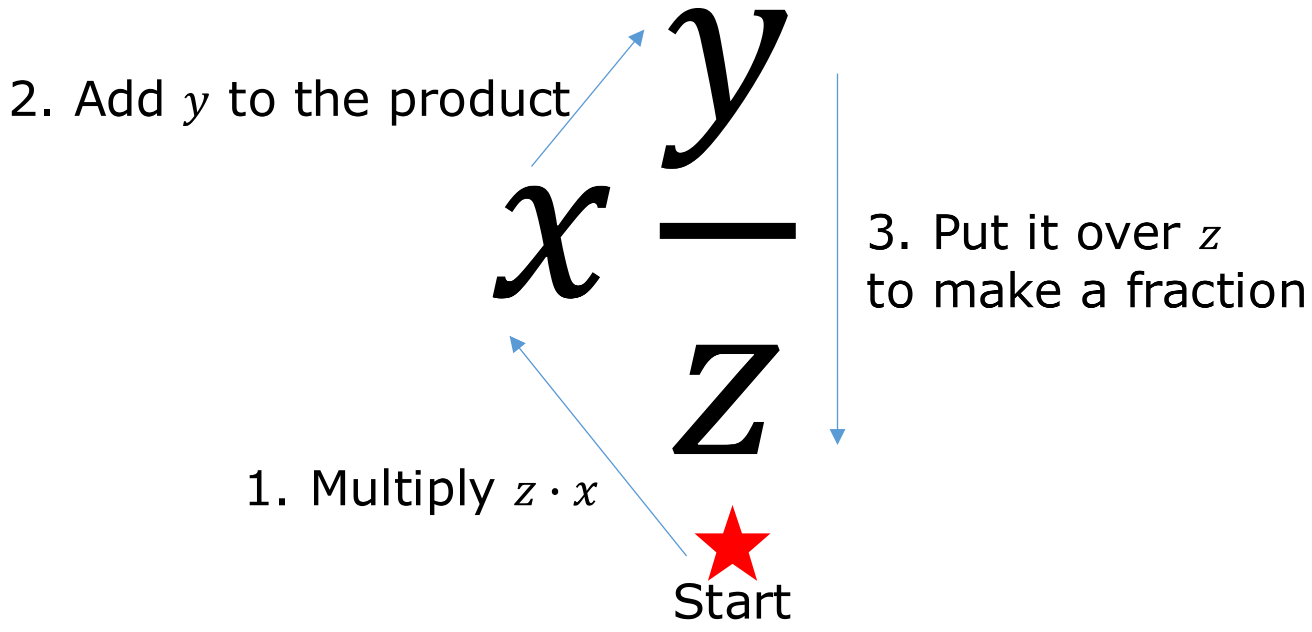
In general, to convert a mixed number into an improper fraction:

$$x\frac{y}{z}$$

1. Multiply the denominator of the fraction (z) by the whole number (x)
2. Add the value of the numerator (y) to the product calculated in Step 1
3. Place this value above the original denominator of the fraction (z)



Converting Mixed Numbers to Improper Fractions



Convert the following Mixed Number into an Improper Fraction:

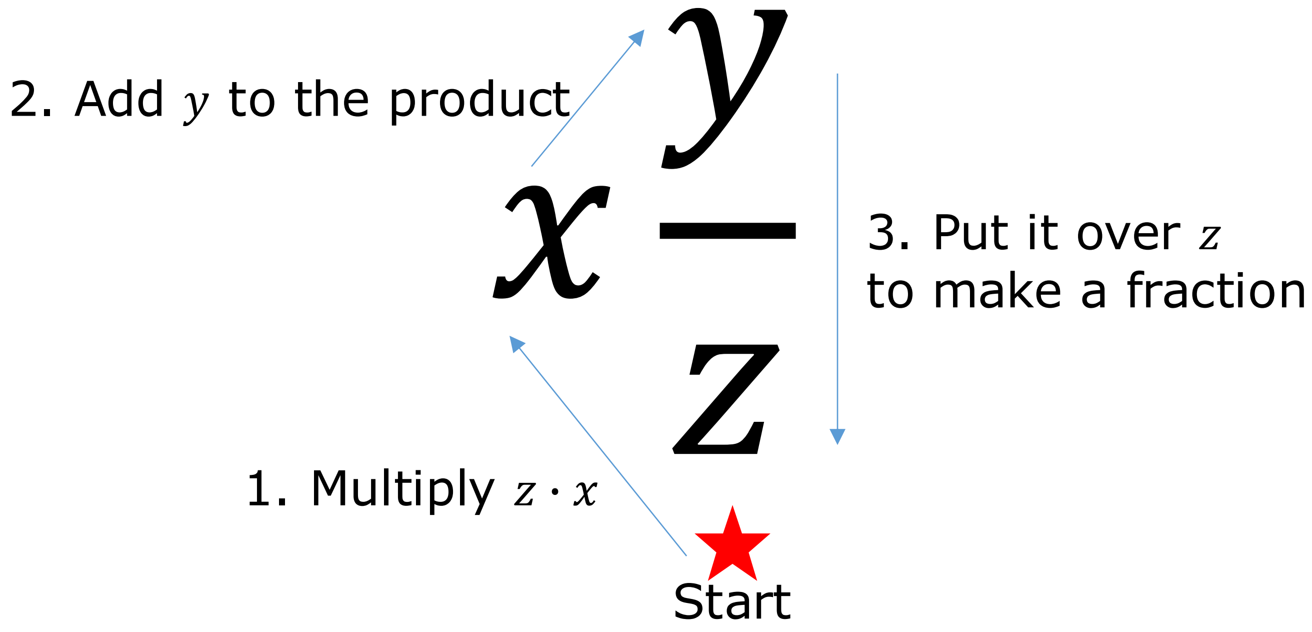
$$5\frac{1}{2}$$

$$2 \cdot 5 = 10$$

$$1 + 10 = 11$$

$$\mathbf{\frac{11}{2}}$$

Converting Mixed Numbers to Improper Fractions



Convert the following Mixed Number into an Improper Fraction:

$$10\frac{3}{8}$$

$$8 \cdot 10 = 80$$

$$3 + 80 = 83$$

$$\mathbf{\frac{83}{8}}$$

Converting Improper Fractions to Mixed Numbers

In general, you must use long division on the improper fraction to convert it into a mixed number

1. Perform long-division on the Improper Fraction

***Remember: the denominator of the improper fraction is always the number on the outside of the long division box; in other words, the denominator is always the divisor

$\frac{x}{y}$ is the same thing as $y\overline{)x}$

$\frac{9}{4}$ is the same thing as $4\overline{)9}$

2.

The **quotient** is the whole number portion of the mixed number

The **remainder** is the numerator of the fraction portion of the mixed number

The **original divisor** / number on the outside of the long-division box is the denominator of the fraction portion of the mixed number

Converting Improper Fractions to Mixed Numbers

In general, you must use long division on the improper fraction to convert it into a mixed number

Convert the following Improper Fraction into a Mixed Number:

$$\frac{7}{3}$$

$$\begin{array}{r|l} & 2 \\ 3 & 7 \\ - & 6 \\ \hline & 1 \end{array}$$

$$\mathbf{2\frac{1}{3}}$$

The **quotient** is the whole number portion of the mixed number

The **remainder** is the numerator of the fraction portion of the mixed number

The **original divisor** / number on the outside of the long-division box is the denominator of the fraction portion of the mixed number

Converting Improper Fractions to Mixed Numbers

In general, you must use long division on the improper fraction to convert it into a mixed number

Convert the following Improper Fraction into a Mixed Number:

$$\frac{38}{5}$$

$$\begin{array}{r} 7 \\ 5 \overline{)38} \\ -35 \\ \hline 3 \end{array}$$

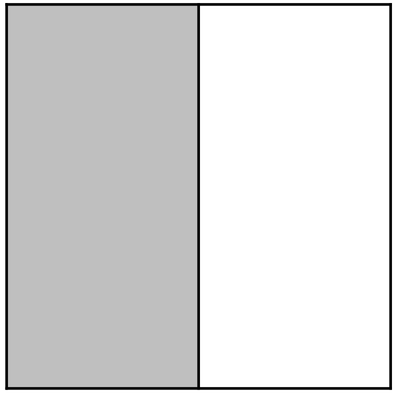
$$\mathbf{7\frac{3}{5}}$$

The **quotient** is the whole number portion of the mixed number

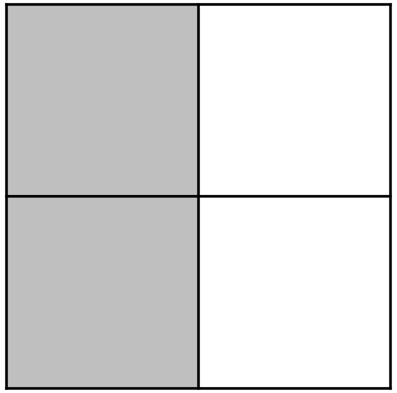
The **remainder** is the numerator of the fraction portion of the mixed number

The **original divisor** / number on the outside of the long-division box is the denominator of the fraction portion of the mixed number

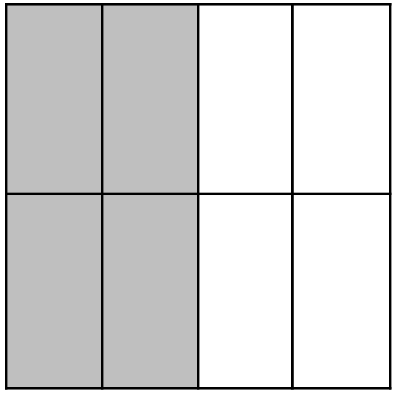
Equivalent Fractions



$$\frac{1}{2}$$



$$\frac{2}{4}$$



$$\frac{4}{8}$$

Fractions that represent the *same* portion of an entire whole are called **equivalent fractions**. All of the above fractions represent the same shaded portion of the square's area, which make them all equivalent fractions. They all represent the exact same thing: **half**

Equivalent Fractions: Testing for Equivalence

The **cross-product** method is used to determine if fractions are equivalent/equal

$$\frac{x}{y} = \frac{w}{z}$$

1. Put the fractions beside each other with an equal sign in between because we are testing for equivalency
2. Multiply the numerator of Fraction 1 by the denominator of Fraction 2 to form a product $(x \cdot z)$
3. Multiply the denominator of Fraction 1 by the numerator of Fraction 2 form a product $(y \cdot w)$
4. Compare the products ---- **Same products: Equivalent** **Different products: Not Equivalent**

Equivalent Fractions: Testing for Equivalence

Determine if the following fractions are equivalent: $\frac{3}{8}$ & $\frac{12}{32}$

$$\frac{3}{8} = \frac{12}{32}$$

$$\begin{array}{r} 32 \\ \times \quad 3 \\ \hline 96 \end{array} \qquad \begin{array}{r} {}^{1} \\ 12 \\ \times \quad 8 \\ \hline 96 \end{array}$$

Since both cross-products are 96, the fractions are equivalent

Equivalent Fractions: Testing for Equivalence

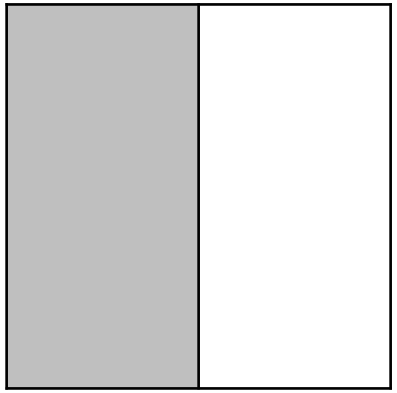
Determine if the following fractions are equivalent: $\frac{3}{4}$ & $\frac{8}{9}$

$$\frac{3}{4} = \frac{8}{9}$$

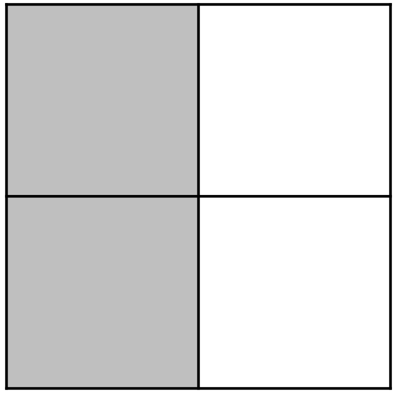
$3 \cdot 9 = \mathbf{27}$ $4 \cdot 8 = \mathbf{32}$

Since both cross-products are different values, the fractions are NOT equivalent

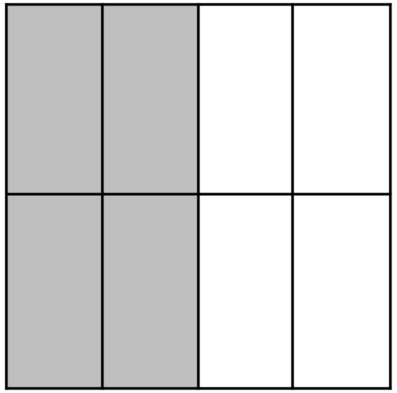
Fractions in Simplest Form



$$\frac{1}{2}$$



$$\frac{2}{4}$$

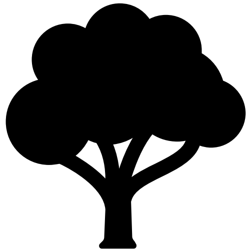


$$\frac{4}{8}$$

Simplest Form: A fraction is written in simplest form (or lowest terms) when the numerator and denominator have *<u>no common factors other than 1</u>*

Unless otherwise stated, always put your answers in simplest form (lowest terms)

Fractions in Simplest Form



In order to write fractions into ***simplest form***, we must use *factor trees* on both the numerator and denominator in order to find what prime factors are *commonly shared*. By finding this, we may ‘cancel out’ its shared factors until 1 is the only shared factor, making it simplified/reduced. *(Remember: 1 is always a commonly shared factor between any two numbers)*

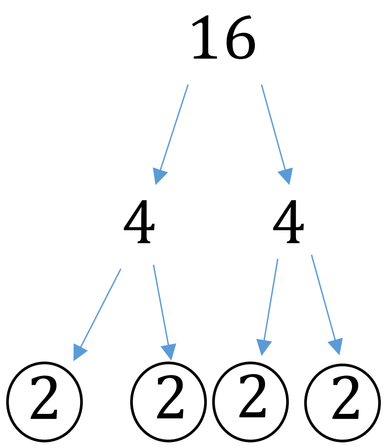
1. Find Prime Factorizations of **numerator** and **denominator**
2. Write the Prime Factorization of the numerator *over* the prime factorization of the denominator in long form (in other words, don’t use exponents because we need to see everything!)
3. Cancel out shared factors from the top (numerator) and bottom (denominator)
4. Multiply the remaining numbers in the numerator, and again in the denominator

Fractions in Simplest Form

Write the following fraction in simplest form:

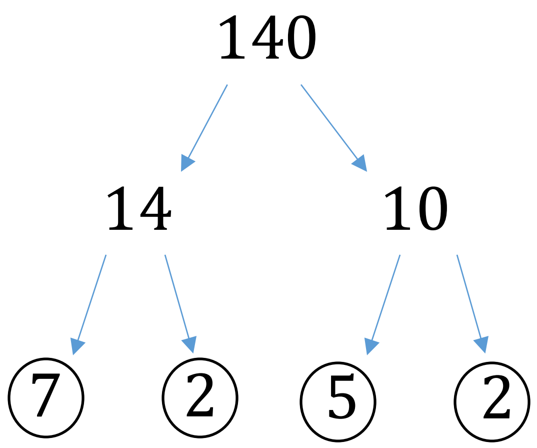
$$\frac{16}{140}$$

Numerator



$$2 \cdot 2 \cdot 2 \cdot 2$$

Denominator



$$2 \cdot 2 \cdot 5 \cdot 7$$

Fractions in Simplest Form

Write the following fraction in simplest form:

$$\frac{16}{140}$$

The Prime Factorization of the **numerator**: $2 \cdot 2 \cdot 2 \cdot 2$

The Prime Factorization of the **denominator**: $2 \cdot 2 \cdot 5 \cdot 7$

$$\frac{\cancel{2} \cdot \cancel{2} \cdot 2 \cdot 2}{\cancel{2} \cdot \cancel{2} \cdot 5 \cdot 7} \rightarrow \frac{4}{35}$$

1. Find Prime Factorizations of **numerator** and **denominator**
2. Write the Prime Factorization of the numerator *over* the prime factorization of the denominator in long form (in other words, don't use exponents because we need to see everything!
3. Cancel out shared factors (if you see a number from the numerator that is the same as a number from the denominator, strike them out)
4. Multiply the remaining numbers in the numerator, and again in the denominator

Fractions in Simplest Form

Write the following fraction in simplest form:

$$\frac{6}{36}$$

Numerator

6

3 2

$2 \cdot 3$

Denominator

36

6 6

2 3 2 3

$2 \cdot 2 \cdot 3 \cdot 3$

Fractions in Simplest Form

Write the following fraction in simplest form:

$$\frac{6}{36}$$

The Prime Factorization of the **numerator**: $2 \cdot 3$

The Prime Factorization of the **denominator**: $2 \cdot 2 \cdot 3 \cdot 3$

$$\frac{\not{2} \cdot \not{3}}{\not{2} \cdot 2 \cdot \not{3} \cdot 3} \rightarrow \frac{1}{6}$$

1. Find Prime Factorizations of **numerator** and **denominator**
2. Write the Prime Factorization of the numerator *over* the prime factorization of the denominator in long form (in other words, don't use exponents because we need to see everything!
3. Cancel out shared factors (if you see a number from the numerator that is the same as a number from the denominator, strike them out)
4. Multiply the remaining numbers in the numerator, and again in the denominator

Multiplying: Fractions & Mixed Numbers

Basic Rule of Multiplying Fractions:

$$\frac{a}{b} \times \frac{c}{d} \Rightarrow \frac{a \cdot c}{b \cdot d}$$

NOTE: THIS IS **NOT** THE SAME METHOD AS FINDING CROSS-PRODUCTS, AS EXPLAINED IN THE PREVIOUS SECTION. **WE ARE NOT TRYING TO DETERMINE EQUIVALENCY SO DO NOT FIND THE CROSS-PRODUCTS.**

When multiplying fractions, multiply straight across:

1. numerator $\times$ numerator
2. denominator $\times$ denominator

Multiplying: Fractions & Mixed Numbers

Multiply:

$$\frac{1}{4} \times \frac{3}{8}$$

$$\frac{a}{b} \times \frac{c}{d} \rightarrow \frac{a \cdot c}{b \cdot d}$$

$$\frac{1}{4} \times \frac{3}{8} = \frac{1 \cdot 3}{4 \cdot 8} = \mathbf{\frac{3}{32}}$$

Multiplying: Fractions & Mixed Numbers

$$\frac{a}{b} \times \frac{c}{d} \Rightarrow \frac{a \cdot c}{b \cdot d}$$

Multiply:

$$\frac{2}{3} \times \frac{2}{7}$$

$$\frac{2}{3} \times \frac{2}{7} = \frac{2 \cdot 2}{3 \cdot 7} = \mathbf{\frac{4}{21}}$$

Multiplying: Fractions & Mixed Numbers

$$\frac{a}{b} \times \frac{c}{d} \Rightarrow \frac{a \cdot c}{b \cdot d}$$

Multiply:

$\frac{2}{3} \times \frac{2}{7}$

$$\frac{2}{3} \times \frac{2}{7} = \frac{2 \cdot 2}{3 \cdot 7} = \frac{4}{21}$$

Both fractions are already in lowest terms because the only commonly shared factors between the numerator and denominator is: **1**

Multiply:

$\frac{1}{4} \times \frac{3}{8}$

$$\frac{1}{4} \times \frac{3}{8} = \frac{1 \cdot 3}{4 \cdot 8} = \frac{3}{32}$$

On the next slide, we will look at an example where numbers in the fractions **do** share other common factors, which means we will have to reduce our solution to simplest form. You will be shown 2 different methods

Multiplying: Fractions & Mixed Numbers

Multiply:

$$\frac{4}{9} \times \frac{18}{20}$$

$$\frac{a}{b} \times \frac{c}{d} \Rightarrow \frac{a \cdot c}{b \cdot d}$$

Method 1: Using Prime Factorizations

Instead of multiplying these out to get $\frac{72}{180}$ then completing factor trees/prime factorizations for 72 and 180 to reduce to simplest form, we can actually achieve our solution in simplest form faster by prime factorizing the ***original*** composite numerators and original composite denominators instead:

$$\frac{2 \cdot 2}{3 \cdot 3} \times \frac{2 \cdot 3 \cdot 3}{2 \cdot 2 \cdot 5} = \frac{\cancel{2} \cdot \cancel{2} \cdot 2 \cdot \cancel{3} \cdot \cancel{3}}{\cancel{2} \cdot \cancel{2} \cdot \cancel{3} \cdot \cancel{3} \cdot 5} = \frac{2}{5}$$

Multiplying: Fractions & Mixed Numbers

Multiply:

$$\frac{4}{9} \times \frac{18}{20}$$

$$\frac{a}{b} \times \frac{c}{d} \rightarrow \frac{a \cdot c}{b \cdot d}$$

Method 2: Dividing by GCF's

Helpful Hint

Greatest Common Factor (GCF): *the largest number that is a common factor between two numbers.*

i.e. **GCF** between 12 and 28:

Factors of 12 → 1, 2, 3, 4, 6, 12

Factors of 28 → 1, 2, 4, 7, 14, 28

The common factors are 1, 2, 4 → ∴ **The GCF is 4**

$$\frac{4}{9} \times \frac{18}{20}$$

Method 2 requires us to find the greatest common factor between one numerator and one denominator. Start with finding the GCF between the numerator of one fraction and the denominator of another fraction.

You **might** need to look within the same fraction as well!

Your main objective in this method: get the numbers as small as possible by dividing by the GCF!

Multiplying: Fractions & Mixed Numbers

Multiply:

$$\frac{4}{9} \times \frac{18}{20}$$

$$\frac{a}{b} \times \frac{c}{d} \rightarrow \frac{a \cdot c}{b \cdot d}$$

GCF between 4 and 20

Factors of 4 → 1, 2, 4

Factors of 20 → 1, 2, 4, 5, 10, 20

The common factors are 1, 2, 4 ∴**The GCF is 4**

GCF between 9 and 18

Factors of 9 → 1, 3, 9

Factors of 18 → 1, 2, 3, 6, 9, 18

The common factors are 1, 3, 9 ∴**The GCF is 9**

Method 2: Dividing by GCF's

Helpful Hint

Greatest Common Factor (GCF): *the largest number that is a common factor between two numbers.*

i.e. **GCF** between 12 and 28:

Factors of 12 → 1, 2, 3, 4, 6, 12

Factors of 28 → 1, 2, 4, 7, 14, 28

The common factors are 1, 2, 4 → ∴ **The GCF is 4**

Method 2 requires us to find the greatest common factor between one numerator and one denominator. Start with finding the GCF between the numerator of one fraction and the denominator of another fraction.

You **might** need to look within the same fraction as well!

Your main objective in this method: get the numbers as small as possible by dividing by the GCF!

Multiplying: Fractions & Mixed Numbers

Multiply:

$$\frac{4}{9} \times \frac{18}{20}$$

$$\frac{a}{b} \times \frac{c}{d} \rightarrow \frac{a \cdot c}{b \cdot d}$$

GCF between 4 and 20: **4**
GCF between 9 and 18: **9**

$$\frac{4 \div 4}{9} \times \frac{18}{20 \div 4} \rightarrow \frac{1}{9} \times \frac{18}{5}$$

Method 2: Dividing by GCF's

Helpful Hint

Greatest Common Factor (GCF): *the largest number that is a common factor between two numbers.*

i.e. **GCF** between 12 and 28:

Factors of 12 → 1, 2, 3, 4, 6, 12

Factors of 28 → 1, 2, 4, 7, 14, 28

The common factors are 1, 2, 4 → ∴ **The GCF is 4**

Method 2 requires us to find the greatest common factor between one numerator and one denominator. Start with finding the GCF between the numerator of one fraction and the denominator of another fraction.

You **might** need to look within the same fraction as well!

Your main objective in this method: get the numbers as small as possible by dividing by the GCF!

Multiplying: Fractions & Mixed Numbers

Multiply:

$$\frac{4}{9} \times \frac{18}{20}$$

$$\frac{a}{b} \times \frac{c}{d} \rightarrow \frac{a \cdot c}{b \cdot d}$$

GCF between 4 and 20: **4**
GCF between 9 and 18: **9**

$$\frac{1}{9 \div 9} \times \frac{18 \div 9}{5} \rightarrow \frac{1}{1} \times \frac{2}{5}$$

Method 2: Dividing by GCF's

Helpful Hint

Greatest Common Factor (GCF): *the largest number that is a common factor between two numbers.*

i.e. **GCF** between 12 and 28:

Factors of 12 → 1, 2, 3, 4, 6, 12

Factors of 28 → 1, 2, 4, 7, 14, 28

The common factors are 1, 2, 4 → ∴ **The GCF is 4**

Method 2 requires us to find the greatest common factor between one numerator and one denominator. Start with finding the GCF between the numerator of one fraction and the denominator of another fraction.

You **might** need to look within the same fraction as well!

Your main objective in this method: get the numbers as small as possible by dividing by the GCF!

Multiplying: Fractions & Mixed Numbers

Multiply:

$$\frac{4}{9} \times \frac{18}{20}$$

$$\frac{a}{b} \times \frac{c}{d} \rightarrow \frac{a \cdot c}{b \cdot d}$$

GCF between 4 and 20: **4**
GCF between 9 and 18: **9**

$$\frac{1}{1} \times \frac{2}{5} = \frac{1 \cdot 2}{1 \cdot 5} = \mathbf{\frac{2}{5}}$$

Method 2: Dividing by GCF's

Helpful Hint

Greatest Common Factor (GCF): *the largest number that is a common factor between two numbers.*

i.e. **GCF** between 12 and 28:

Factors of 12 → 1, 2, 3, 4, 6, 12

Factors of 28 → 1, 2, 4, 7, 14, 28

The common factors are 1, 2, 4 → ∴ **The GCF is 4**

Method 2 requires us to find the greatest common factor between one numerator and one denominator. Start with finding the GCF between the numerator of one fraction and the denominator of another fraction.

You **might** need to look within the same fraction as well!

Your main objective in this method: get the numbers as small as possible by dividing by the GCF!

Multiplying: Fractions & Mixed Numbers

Multiply:

Method 1: Using Prime Factorizations

$$\frac{21}{35} \times \frac{10}{28}$$

$$\frac{3 \cdot 7}{5 \cdot 7} \times \frac{2 \cdot 5}{2 \cdot 2 \cdot 7} = \frac{3 \cdot \cancel{7} \cdot \cancel{2} \cdot \cancel{5}}{\cancel{5} \cdot \cancel{7} \cdot \cancel{2} \cdot 2 \cdot 7} = \frac{3}{2 \cdot 7} = \mathbf{\frac{3}{14}}$$

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Multiplying: Fractions & Mixed Numbers

Method 2: Dividing by GCF's

Multiply:

$$\frac{21}{35} \times \frac{10}{28}$$

GCF between 21 and 28: **7**
GCF between 35 and 10: **5**

$$\frac{21 \div 7}{35 \div 5} \times \frac{10 \div 5}{28 \div 7}$$

$$\frac{3}{7} \times \frac{2}{4}$$

At this point, the 2 and 4 still share common factors.

The GCF between 2 and 4 is **2**, so divide both by **2**

$$\frac{3}{7} \times \frac{2 \div 2}{4 \div 2}$$

$$\frac{3}{7} \times \frac{1}{2} = \frac{3 \cdot 1}{2 \cdot 7} = \mathbf{\frac{3}{14}}$$

Multiplying: Fractions & Mixed Numbers

Multiply:

$\frac{4}{5} \times 100$

Method 1: Using Prime Factorizations

"100" is not a fraction… so how do we make it into one?

100 expressed as a fraction: $\frac{100}{1}$ because $\frac{100}{1} = 100$

When working with fractions, whenever you see a 'floating' whole number (whether before or after a $\times$ or $\div$), put that number over 1 as it is just another way of representing that whole number, as a fraction

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Multiplying: Fractions & Mixed Numbers

Multiply:

$\frac{4}{5} \times 100$

Method 1: Using Prime Factorizations

$$\frac{4}{5} \times \frac{100}{1}$$

$$\frac{2 \cdot 2}{5} \times \frac{2 \cdot 2 \cdot 5 \cdot 5}{1} = \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot \cancel{5} \cdot 5}{\cancel{5} \cdot 1}$$

$$\frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 5}{1} = \frac{80}{1} = \mathbf{80}$$

Multiplying: Fractions & Mixed Numbers

Multiply:

$\frac{4}{5} \times 100$

Method 2: Dividing by GCF's

$$\frac{4}{5} \times \frac{100}{1}$$

The only numbers that share common factors are **100** & **5**

The GCF between 100 and 5 is: **5**

$$\frac{4}{5 \div 5} \times \frac{100 \div 5}{1}$$

$$\frac{4}{1} \times \frac{20}{1} = \frac{4 \cdot 20}{1 \cdot 1} = \frac{80}{1} = \mathbf{80}$$

Multiplying: Fractions & Mixed Numbers

Multiply:

$$\frac{12}{14} \times \frac{5}{24} \times \frac{22}{45}$$

Method 1: Using Prime Factorizations

$$\frac{2 \cdot 2 \cdot 3}{2 \cdot 7} \times \frac{5}{2 \cdot 2 \cdot 2 \cdot 3} \times \frac{2 \cdot 11}{3 \cdot 3 \cdot 5}$$

$$\frac{\cancel{2} \cdot \cancel{2} \cdot \cancel{3} \cdot \cancel{5} \cdot \cancel{2} \cdot 11}{\cancel{2} \cdot 7 \cdot \cancel{2} \cdot \cancel{2} \cdot 2 \cdot \cancel{3} \cdot 3 \cdot 3 \cdot \cancel{5}}$$

$$\frac{11}{7 \cdot 2 \cdot 3 \cdot 3}$$

$$= \mathbf{\frac{11}{126}}$$

Multiplying: Fractions & Mixed Numbers

Method 2: Dividing by GCF's

Multiply:

$$\frac{12}{14} \times \frac{5}{24} \times \frac{22}{45}$$

$$\frac{12 \div 12}{14 \div 2} \times \frac{5 \div 5}{24 \div 12} \times \frac{22 \div 2}{45 \div 5}$$

$$\frac{1}{7} \times \frac{1}{2} \times \frac{11}{9}$$

$$\frac{1 \cdot 1 \cdot 11}{7 \cdot 2 \cdot 9} = \mathbf{\frac{11}{126}}$$

Multiplying: Fractions & Mixed Numbers

Estimate then Solve:

$$3\frac{1}{5} \times 4\frac{5}{8}$$

Estimate

$3\frac{1}{5} \rightarrow 3$

$4\frac{5}{8} \rightarrow 5$

$3 \times 5 = \mathbf{15}$

*** Before multiplying, convert ANY mixed numbers into **improper fractions**

Actual

$3\frac{1}{5} \rightarrow \frac{16}{5}$

$4\frac{5}{8} \rightarrow \frac{37}{8}$

$$\frac{\overset{2}{\cancel{16}}}{5} \times \frac{37}{\underset{1}{\cancel{8}}} = \frac{2 \cdot 37}{5 \cdot 1} = \frac{74}{5}$$

$$\begin{array}{r} 14 \\ 5\overline{)74} \\ -\underline{5} \\ 24 \\ -\underline{20} \\ 4 \end{array}$$

$$= \mathbf{14\frac{4}{5}}$$

Dividing Fractions

- The "Reciprocal"

Before we begin to divide fractions, we need to know how to find the **reciprocal** of a fraction or whole number

*** Two numbers are reciprocals of each other if their resulting *product* = 1

Multiply!

The reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$ because: $\frac{a}{b} \cdot \frac{b}{a} = \frac{\cancel{a} \cdot \cancel{b}}{\cancel{b} \cdot \cancel{a}} = 1$

Dividing Fractions

- The "Reciprocal"

Before we begin to divide fractions, we need to know how to find the **reciprocal** of a fraction or whole number

*** Two numbers are reciprocals of each other if their resulting *product* = 1

The reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$ because: $\frac{3}{4} \cdot \frac{4}{3} = \frac{3 \cdot 4}{4 \cdot 3} = \frac{12}{12} = 1$

Dividing Fractions

- The "Reciprocal"

Before we begin to divide fractions, we need to know how to find the **reciprocal** of a fraction or whole number

*** Two numbers are reciprocals of each other if their resulting *product* = 1

The reciprocal of 7 is $\frac{1}{7}$ because: $$\frac{7}{1} \cdot \frac{1}{7} = \frac{7 \cdot 1}{1 \cdot 7} = \frac{7}{7} = 1$$

Dividing: Fractions & Mixed Numbers

Dividing fractions/numbers is the same thing as <u>multiplying</u> the:

first original fraction/number $\times$ **the reciprocal of the second fraction/number**

BUT WHY?

$$10 \div 5 = \mathbf{2}$$

Consider the following expression:

$$10 \div 5$$

$$10 \times \frac{1}{5}$$

$$\frac{\overset{2}{\cancel{10}}}{1} \times \frac{1}{\underset{1}{\cancel{5}}}$$

$$\frac{2 \cdot 1}{1 \cdot 1} = \frac{2}{1}$$

$$= \mathbf{2}$$

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Dividing: Fractions & Mixed Numbers

Basic Rule of Dividing Fractions:

$$\frac{a}{b} \div \frac{c}{d} \Rightarrow \frac{a}{b} \times \frac{d}{c}$$

When dividing fractions,

1) Keep the first original fraction the same (the fraction before the "÷")
2) Drop the "÷" and replace with "×"
3) **FLIP** the second original fraction (this is the **reciprocal**!)
4) *Multiply* as per usual

Dividing: Fractions & Mixed Numbers

Dividing fractions/numbers is the same thing as <u>multiplying</u> the:

first original fraction/number × the reciprocal of the second fraction/number

Straight Division:

$20 \div 2 = \mathbf{10}$

What is $\frac{1}{2}$ of 20 ?

As an expression:

$20 \div 2$

Multiplying by the reciprocal:

$$20 \times \frac{1}{2}$$

$$\frac{\overset{10}{\cancel{20}}}{1} \times \frac{1}{\underset{1}{\cancel{2}}}$$

$$\frac{10}{1} \times \frac{1}{1} = \frac{10 \cdot 1}{1 \cdot 1} = \frac{10}{1}$$

$$= \mathbf{10}$$

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Dividing: Fractions & Mixed Numbers

Dividing fractions/numbers is the same thing as <u>multiplying</u> the:

first original fraction/number $\times$ **the reciprocal of the second fraction/number**

Straight Division:

$28 \div 4 = \mathbf{7}$

What is $\frac{1}{4}$ of 28?

As an expression:

$28 \div 4$

Multiplying by the reciprocal:

$$28 \times \frac{1}{4}$$

$$\frac{\overset{7}{\cancel{28}}}{1} \times \frac{1}{\underset{1}{\cancel{4}}}$$

$$\frac{7}{1} \times \frac{1}{1} = \frac{7 \cdot 1}{1 \cdot 1} = \frac{7}{1}$$

$$= \mathbf{7}$$

Dividing: Fractions & Mixed Numbers

Dividing fractions/numbers is the same thing as <u>multiplying</u> the:

first original fraction/number $\times$ **the reciprocal of the second fraction/number**

Multiplying by the reciprocal:

What is half of $\frac{1}{2}$?

As an expression:

$$\frac{1}{2} \div 2$$

$$\frac{1}{2} \div 2$$

$$\frac{1}{2} \times \frac{1}{2}$$

$$\frac{1 \cdot 1}{2 \cdot 2}$$

$$= \mathbf{\frac{1}{4}}$$

The reciprocal of 2 is: $\frac{1}{2}$

Dividing: Fractions & Mixed Numbers

Dividing fractions/numbers is the same thing as <u>multiplying</u> the:

first original fraction/number × **the reciprocal of the second fraction/number**

Divide and simplify:

$$\frac{7}{8} \div \frac{2}{9}$$

$$\frac{7}{8} \times \frac{9}{2}$$

There are no shared factors so multiply through

$$\frac{7 \cdot 9}{8 \cdot 2}$$

$$\mathbf{\frac{63}{16}}$$

OR

$$\begin{array}{r} 3 \\ 16\overline{)63} \\ -\underline{48} \\ 15 \end{array}$$

$$= \mathbf{3\frac{15}{16}}$$

Dividing: Fractions & Mixed Numbers

Dividing fractions/numbers is the same thing as <u>multiplying</u> the:

first original fraction/number $\times$ **the reciprocal of the second fraction/number**

Divide and simplify:

$$\frac{5}{16} \div \frac{3}{4}$$

$$\frac{5}{\cancel{16}_{\,4}} \times \frac{\cancel{4}^{\,1}}{3}$$

$$\frac{5}{4} \times \frac{1}{3}$$

$$\frac{5 \cdot 1}{4 \cdot 3} = \mathbf{\frac{5}{12}}$$

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Dividing: Fractions & Mixed Numbers

Dividing fractions/numbers is the same thing as <u>multiplying</u> the:

first original fraction/number $\times$ **the reciprocal of the second fraction/number**

Divide and simplify:

$$1\frac{4}{9} \div 2\frac{5}{6}$$

$$\frac{13}{9} \div \frac{17}{6}$$

$$\frac{13}{\cancel{9}_{\,3}} \times \frac{\cancel{6}^{\,2}}{17}$$

$$\frac{13}{3} \times \frac{2}{17} = \frac{13 \cdot 2}{3 \cdot 17} = \mathbf{\frac{26}{51}}$$

The People of Foundational Literacy: Winston Swappie

Winston is a 60 year old culinary chef of Naskapi descent who lives in Montreal, Quebec. He is currently the head chef of Le Café de la Maison Ronde, a popular spot in the heart of the city.

Today, he is applying some newly learned math concepts into the work he does everyday so that he may continue to find success in the hospitality industry in a post-pandemic world.

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Dividing Fractions: Problem Solving

Winston is currently prepping ground beef into patties for his restaurant's new burger menu launching this month. He has a total of $27\frac{3}{4}$ lbs. of ground beef that he needs to make into $\frac{1}{4}$ lb. patties.

How many $\frac{1}{4}$ lb. patties will Winston be able to prep today?

$27\frac{3}{4}$ total lbs of ground beef

$\frac{1}{4}$ lb. patties

Dividing Fractions: Problem Solving

Winston is currently prepping ground beef into patties for his restaurant's new burger menu launching this month. He has a total of $27\frac{3}{4}$ lbs. of ground beef that he needs to make into $\frac{1}{4}$ lb. patties.

How many $\frac{1}{4}$ lb. patties will Winston be able to prep today?

the # of patties Winston will be able to prep	=	Total amount of ground beef (lbs)	÷	Intended burger size (lbs)
		$27\frac{3}{4}$		$\frac{1}{4}$

Dividing Fractions: Problem Solving

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Winston is currently prepping ground beef into patties for his restaurant's new burger menu launching this month. He has a total of $27\frac{3}{4}$ lbs. of ground beef that he needs to make into $\frac{1}{4}$ lb. patties.

How many $\frac{1}{4}$ lb. patties will Winston be able to prep today?

the # of patties Winston will be able to prep $=$ Total amount of ground beef (lbs) $\div$ Intended burger size (lbs)

$$27\frac{3}{4} \div \frac{1}{4}$$

$$\frac{111}{4} \div \frac{1}{4}$$

$$\frac{111}{\cancel{4}_{1}} \times \frac{\cancel{4}^{1}}{1} = \frac{111}{1} \times \frac{1}{1} = \frac{111 \cdot 1}{1 \cdot 1} = \frac{111}{1}$$

$$= 111$$

ANSWER the question in sentence form: With $27\frac{3}{4}$ lbs of ground beef, Winston will be able to prep 111 ($\frac{1}{4}$ lb. sized) burgers.

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