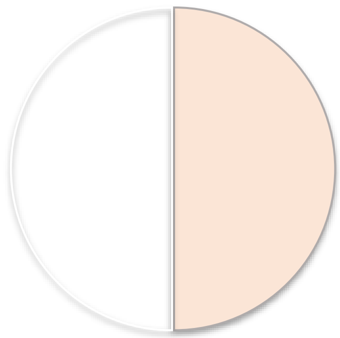
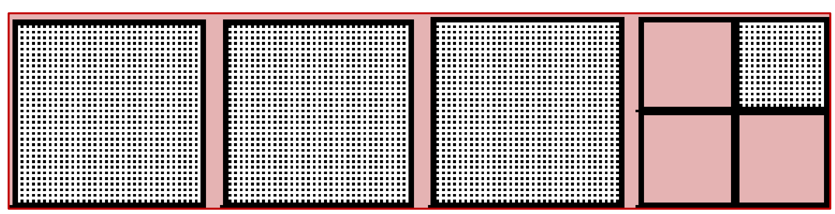
$\frac{1}{2}$



Foundational Numeracy

$3\frac{1}{4}$



Module 5: Fractions

Facilitator Guide

V1.18 – 04/June/18

Developed for Alberta's Community Adult Learning Program

Funded by Alberta Advanced Education

NorQuest College
10215 108 Street NW,
Edmonton AB, T5J 1L6
Attn: Curriculum Development

Contents

Introduction to the Module

In Applied Numeracy – Advanced, you will explore foundational concepts and key terms used in math and science in order to successfully transition into Academic Upgrading. This module is a quick review of fractions. Enjoy your studies!

Important

When you see an object like the one below, you can use a QR code scanner on your phone or tablet, and it will play a video of the math example.

Scan me

Want to watch a video of this lesson?
Scan the QR Code to the left,
or use the link below:
https://youtu.be/bFosWzDPALg

Note: The facilitator guide mirrors the Learner Guide with a couple of key differences.

- Facilitator notes throughout the module in boxes like this. Include teaching strategies and common errors
- Student Practice doesn't have this bubble. Instructor led The instructor can teach the concept or the learner can watch the video

Specific Learning Outcomes

The table below displays the skills and knowledge you will explore in this module. This is your opportunity to evaluate your own skills to see if you can do these things. At the end of this module, you will be invited to re-evaluate your skills to measure the progress you have made.

In this module, I will learn how to …	I can't do this	I can do this with help	I can do this!
1. Use fractions in everyday math; solve fraction-related problems			
2. Add and subtract all types of fractions			
3. Multiply and divide all types of fractions			

Essential Skills

Numeracy: Using and understanding numbers

Thinking: Finding information, solving problems, making decisions, using your memory, planning job tasks and being organized, thinking critically

Vocabulary: Gaining related vocabulary

Unit 1: Introduction to Fractions

Think about …

✦ Baking and cooking include parts and portions of ingredients. You may read in your recipe something like ⅓ cup of sugar or ⅔ cup butter. Have you ever thought about what these measurements really mean?

Keywords

Denominator	Lower (bottom) part of a fraction; tells how many equal parts a whole has been divided into
Fraction	One or more equal parts of anything
Numerator	Upper (top) part of a fraction; tells how many parts are being considered

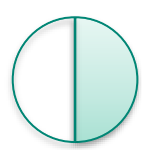
- Learners will sometimes use top and bottom. Try to get them to use the proper terminology numerator (top) and denominator (bottom
- $\frac{\text{numerator}}{\text{denominator}}$ Learners see **Introduction to fractions** https://youtu.be/bFosWzDPALg

What is a fraction?

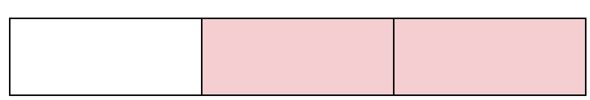
A fraction is one or more equal parts of anything. Fractions have two terms, an upper term called the **numerator** and a lower term called the **denominator**. The denominator tells how many **equal** parts a whole has been divided into. The numerator tells how many of these parts are being considered.

Examples:

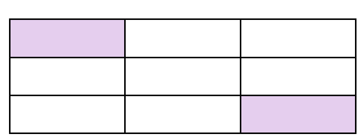
1. $\frac{1}{2}$ ← numerator / ← denominator   1 of 2 **equal** parts



2. $\frac{2}{3}$ 2 of 3 **equal** parts



3. $\frac{2}{9}$ 2 of 9 **equal** parts



Exercise 1.1

1. $\frac{1}{4}$ is called "one fourth" or "one quarter." It means 1 of 4 equal parts.

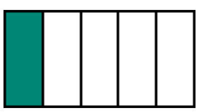
2. $\frac{1}{3}$ is called "______________________________." It means ____ of ____ equal parts.

3. $\frac{1}{6}$ is called "______________________________." It means ____ of ____ equal parts.

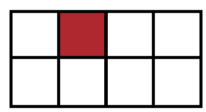
4. $\frac{3}{4}$ is called "______________________________." It means ____ of ____ equal parts.

5. $\frac{2}{3}$ is called "______________________________." It means ____ of ____ equal parts.

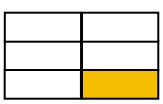
6. What part of the whole is shaded?

a. 

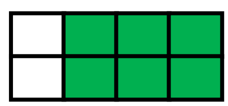
____ of ____ equal parts

b. 

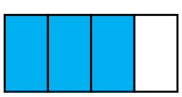
____ of ____ equal parts

c. 

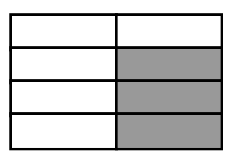
____ of ____ equal parts

d. 

____ of ____ equal parts

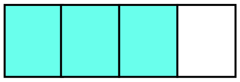
e. 

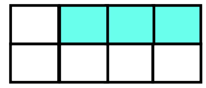
____ of ____ equal parts

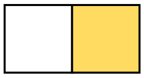
f. 

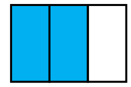
____ of ____ equal parts

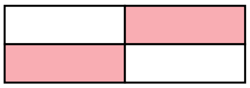
7. What fraction of the whole do the shaded parts represent?

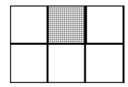
a.  ________

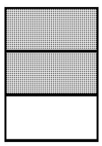
b.  ________

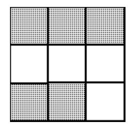
c.  ________

d.  ________

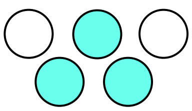
e.  ________

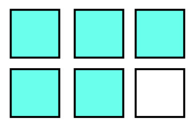
f.  ________

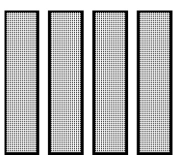
g.  ________

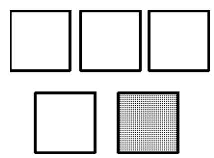
h.  ________

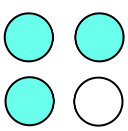
8. What fraction of each group is shaded?

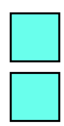
a.  ________

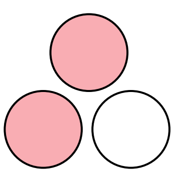
b.  ________

c.  ________

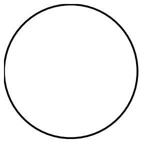
d.  ________

e.  ________

f.  ________

g.  ________

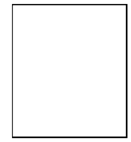
9. Use a pencil and ruler to divide each figure into equal parts as indicated by the denominator. Shade the appropriate number of parts to represent the given fraction.

a. 

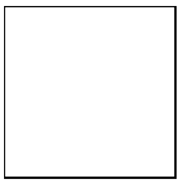
$\frac{1}{2}$

b. 

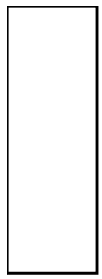
$\frac{7}{8}$

c. 

$\frac{2}{3}$

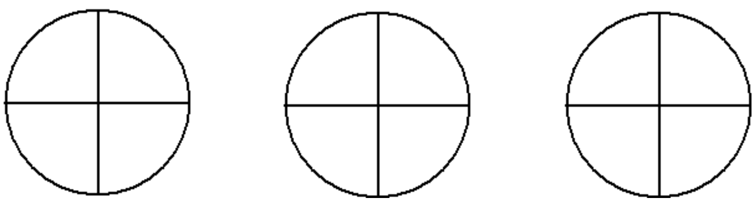
d. 

$\frac{3}{5}$

e. 

$\frac{4}{4}$

10. Four students equally share three pizzas. What fraction does each student get? ___________



11. Draw a square.

a. Divide it in half. What fraction is each part? ________________

b. Divide each of the parts in half. What fraction is each part now? ________________

c. Divide each of these parts in half. What fraction is each part now? ________________

d. Divide each of these parts in half. What fraction is each part now? ________________

e. As the pieces get smaller, what happens to the denominator of each fraction? Why?

____

____

12. Tony received 12 CDs in the mail. He sent 4 of the CDs back. What fraction of the CDs did he keep? (**Hint:** Draw a picture to help, if needed.)

Types of Fractions

- Proper fractions mean less that one
- Improper fractions mean one or more than one.
- Mixed numbers have a whole number and a fractional part.
- Learner will see Introductory Video **https://youtu.be/17IgK9b6P2M**

There are three types of fractions: **proper** fractions, **improper** fractions, and **mixed** numbers.

Examples:

1. **Proper fractions** are fractions in which the numerator is **less than** the denominator.

   $\frac{1}{2}, \frac{2}{3}, \frac{4}{5}, \frac{7}{90}$ are examples of proper fractions.

2. **Improper fractions** are fractions in which the numerator is **greater than** or **equal to** the denominator.

   $\frac{8}{5}, \frac{9}{4}, \frac{4}{3}, \frac{11}{6}, \frac{5}{5}$ are examples of improper fractions.

3. **Mixed numbers** are a combination of a whole number and a proper fraction.

   $3\frac{1}{2}, 5\frac{7}{16}, 4\frac{1}{8}, 9\frac{2}{3}$ are examples of mixed numbers.

Exercise 1.2

Name each of the following as a **proper** fraction, **improper** fraction, or a **mixed number**.

1. $\frac{7}{8}$ ________________
2. $\frac{9}{6}$ ________________
3. $1\frac{4}{6}$ ________________
4. $\frac{4}{4}$ ________________
5. $\frac{5}{4}$ ________________
6. $\frac{3}{2}$ ________________
7. $2\frac{1}{4}$ ________________
8. $\frac{17}{6}$ ________________
9. $\frac{18}{8}$ ________________
10. $12\frac{5}{8}$ ________________
11. $\frac{7}{6}$ ________________
12. $\frac{6}{7}$ ________________

Mixed Numbers and Improper Fractions

Introductory Video

Want to watch a video of this lesson?
Scan the QR Code to the left,
or use the link below:
https://youtu.be/qk2oP6FZ6HA

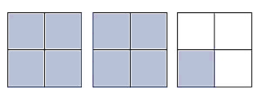
Scan me

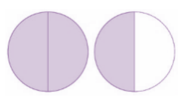
Mixed numbers can be written as improper fractions and improper fractions can be written as mixed numbers.

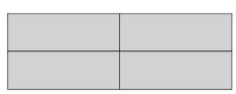
Mixed numbers and improper fractions mean the same amount. They are just shown in a different way. Example $1\frac{3}{4}=\frac{7}{4}$ When you think of money if you have 7 quarters in your wallet, you have one dollar seventy five or 1 looney and 3 quarters.

Fraction strips are manipulatives that can be used to demonstrate converting mixed numbers to improper fractions and improper fractions to mixed numbers. If you don't have fraction strips available drawing pictures works as well.

Examples:

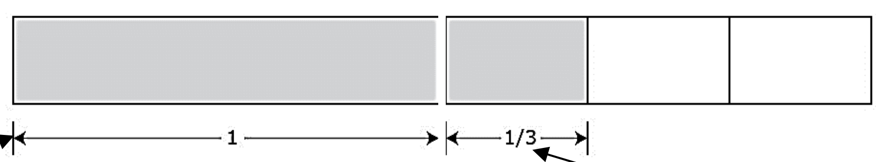
1. $2\frac{1}{4}$ may be represented by  but could also be written as $\frac{9}{4}$.

2. $\frac{3}{2}$ may be represented by  or written as the mixed number $1\frac{1}{2}$.

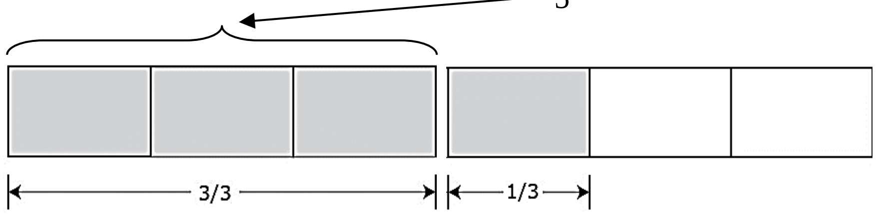
3. $\frac{4}{4}$ may be represented by  or as the whole number 1.

4. Fraction strips can also be used.

$1\frac{1}{3}$ is represented by



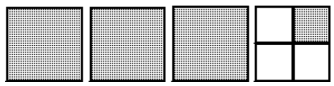
The 1 strip can be replaced with the strip $\frac{3}{3}$. (Use same **equal** parts as the fraction provided.)



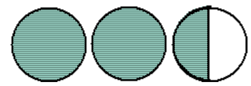
Therefore, $1\frac{1}{3} = \frac{4}{3}$.

Exercise 1.3

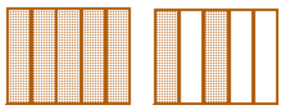
1. Write the mixed number **and** the improper fraction that represent the shaded portions of each diagram.

a. 

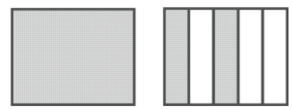
________ and ________

b. 

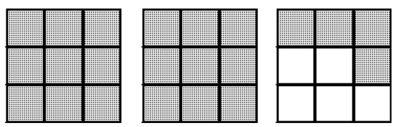
________ and ________

c. 

________ and ________

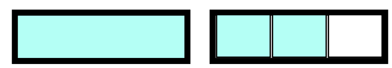
d. 

________ and ________

e. 

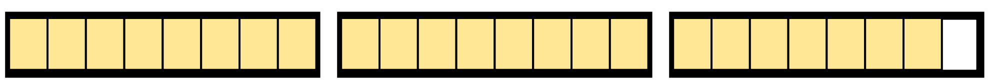
________ and ________

f. Fraction strips:



________ and ________

g. Fraction strips:



________ and ________

2. Draw a diagram to represent each improper fraction or mixed number.

a. $1\frac{3}{5}$

b. $\frac{8}{5}$

c. $3\frac{1}{2}$

d. $\frac{7}{2}$

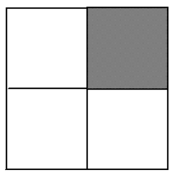
e. $1\frac{1}{4}$

f. $\frac{5}{4}$

Equivalent Fractions

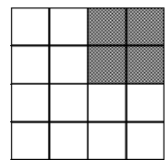
Two fractions are **equivalent** when they represent the **same part** of a whole. For example, ½ a dozen eggs has the same meaning as $\frac{6}{12}$ of a dozen eggs.

The diagrams below show pictures of equivalent fractions. Notice in the diagrams that a fraction can be raised to higher terms by increasing the number of equal parts, or reduced to lower terms by combining equal parts.



$= \frac{1}{4}$

Square A

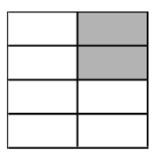


$= \frac{4}{16}$

Square B

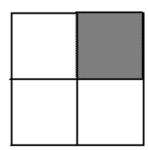
Both of the large squares A and B are the same size, and the area shaded on both squares is the same size. In Square B, the number of equal parts was increased.

What we are doing is multiplying the numerator and denominator by 4 $\frac{1}{4} = \frac{1 \times 4}{4 \times 4} = \frac{4}{16}$.



$= \frac{2}{8}$

Square A



$= \frac{1}{4}$

Square B

In Square A, equal parts were combined. Therefore, $\frac{2}{8} = \frac{1}{4}$. What we are doing is dividing the numerator and denominator by 2 $\frac{2}{8} = \frac{2 \div 2}{8 \div 2} = \frac{1}{4}$.

Student Practice

Want to watch a video of this lesson?
Scan the QR Code to the left,
or use the link below:
https://youtu.be/U2ovEuEUxXQ

Scan me

Make equivalent fractions with the stated denominator

1. $\frac{1}{2} = \frac{}{4}$

2. $\frac{1}{2} = \frac{}{8}$

3. $\frac{3}{5} = \frac{}{35}$

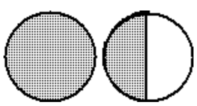
Changing Improper Fractions to Mixed Numbers or Whole Numbers

- Teach students to show long division. The student will need to understand, which is the whole number, which is the remainder and which is the denominator
- Common errors - 1. Students are unsure which numbers go where in the mixed number.
  2. Trying to make a mixed number from a proper fraction. (Not possible)

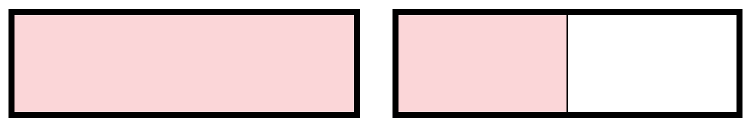
$\frac{7}{4} = 7 \div 4 = 1\frac{3}{4}$

$\frac{4}{7}$ *not possible* Learner sees Introductory video https://youtu.be/-imFslMIN1g

The shaded part of the diagram below can be represented by

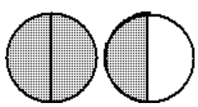


$1\frac{1}{2}$

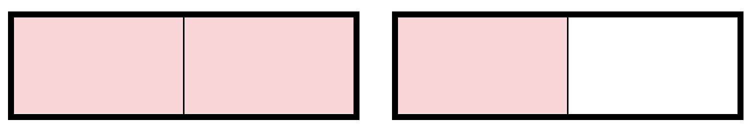


Replace the 1's strip with $\frac{2}{2}$

or by



$\frac{3}{2}$



It is important to know that improper fractions and mixed numbers are written differently but mean exactly the same thing. So, it is possible to change an improper fraction to a mixed number or to change a mixed number to an improper fraction.

To complete many operations, improper fractions must be changed to mixed numbers. This procedure involves division.

Example:

$$\frac{26}{3} = 3\overline{)26}\ \ (8,\ 24,\ 2) = 8\frac{2}{3}$$

Student practice: Instructor led

$\frac{7}{4} =$

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Exercise 1.4

Change the following improper fractions to mixed numbers or whole numbers.

1. $\frac{3}{2}$
2. $\frac{4}{3}$
3. $\frac{27}{5}$
4. $\frac{30}{6}$
5. $\frac{33}{7}$
6. $\frac{17}{3}$
7. $\frac{21}{4}$
8. $\frac{14}{2}$

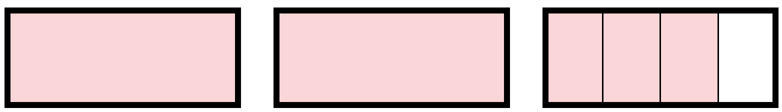
Changing Mixed Numbers to Improper Fractions

- Teach students to work clockwise. Start at the denominator and multiply with the whole number then add the numerator. **Emphasize the denominator doesn't change**

$1\frac{3}{4} = \frac{4\times1+3}{4} = \frac{7}{4}$ Learner sees Intro video https://youtu.be/shpf9krdXQQ

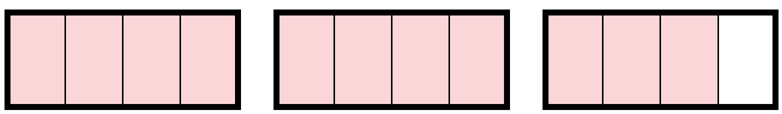
Recall that the shaded part of the diagram below can be represented by

$2\frac{3}{4}$

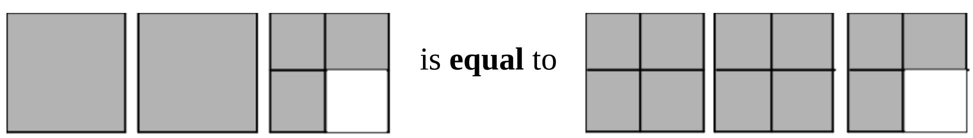


Replace each 1's strip with $\frac{4}{4}$.

or $\frac{11}{4}$



or



is **equal** to

$2\frac{3}{4} = \frac{11}{4}$

This process of changing mixed numbers to improper fractions is used when multiplying or dividing fractions.

Example:

1. $4\frac{3}{5} = \frac{5\times4+3}{5} = \frac{23}{5}$ To change a mixed number to an improper fraction, multiply the denominator by the whole number and add the numerator.

Student practice:

1. $5\frac{1}{4}$

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To change any **whole number** to an **improper fraction**, write the whole number over 1.

Examples:

1. $4 = \frac{4}{1}$
2. $12 = \frac{12}{1}$
3. $17 = \frac{17}{1}$
4. $3 = \frac{?}{4}$ $\quad 3 = \frac{3}{1}$ Therefore, to change the denominator to a 4, you need to multiply **both** the numerator and denominator by 4.

$$\frac{3}{1} = \frac{3 \times 4}{1 \times 4} = \frac{12}{4}$$

Exercise 1.5

1. Change the following to improper fractions.

a. $2\frac{4}{5} =$

b. $8\frac{1}{4} =$

c. $8\frac{3}{8} =$

d. $7 = \frac{?}{3}$

e. $7\frac{2}{3} =$

f. $8 = \frac{?}{2}$

2. Identify each of the following as an improper fraction or a mixed number. Change improper fractions to mixed numbers and mixed numbers to improper fractions.

a. $3\frac{3}{4} =$

____

b. $\frac{13}{3} =$

____

c. $8\frac{3}{7} =$

____

d. $7\frac{2}{3} =$

____

e. $\frac{8}{5} =$ ____________________________

f. $9\frac{3}{4} =$ ____________________________

3. Andrew organizes a delivery of flyers every Saturday. He packs them in bags of 10. Last week, he packed 253 flyers. How many bags did he pack? Give your answer as an improper fraction and as a mixed number.

4. The Earth turns on its axis once every 24 hours. How many turns does it complete in each of the following lengths of time? Express each answer as an improper fraction and a mixed number.

   a. 25 hours

   b. 55 hours

5. The Bentley family drove for 197 minutes to reach the lake. Write 197 minutes as a mixed number of hours.

Locating a Fraction on a Number Line

Introductory Video

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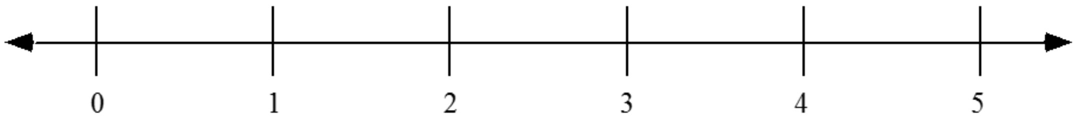
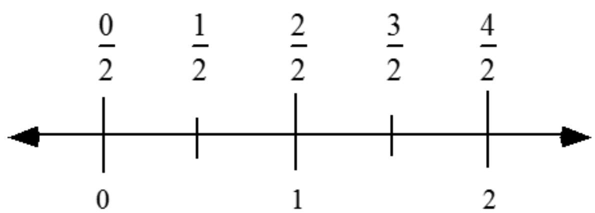
A number line is a line that is labelled with a scale. That is, numbers mark the line at equal intervals.



A number line is endless. It goes to infinity in both directions. Only part of a number line is shown here. Each point on the line is represented by a number. Notice that the whole numbers are **equally spaced** along the line. The interval between whole numbers may be divided into halves, thirds, fourths, etc. In this way, the number line may be related to fractions.

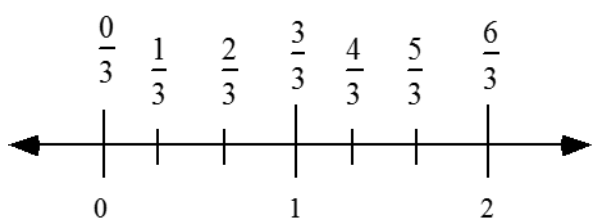
The examples below show the location of various fractions on a number line.

1.



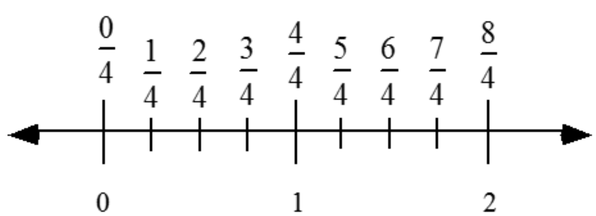
Each whole unit has been divided into halves (2 equal parts).

2.



Each whole has been divided into thirds (3 equal parts).

3.



Each whole unit has been divided into fourths (4 equal parts).

The location of any fraction may be shown on a number line by dividing each whole into the appropriate number of equal parts. For example, to illustrate tenths, divide each whole into 10 equal parts.

Student practice:

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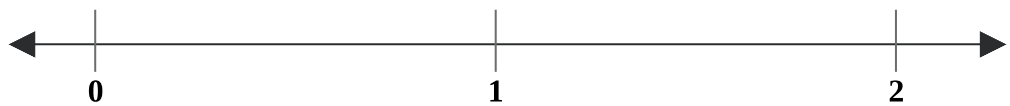
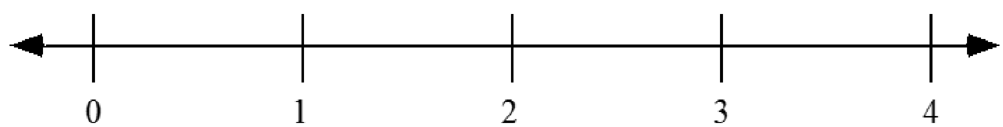
Place $\frac{2}{5}$ on the number line.



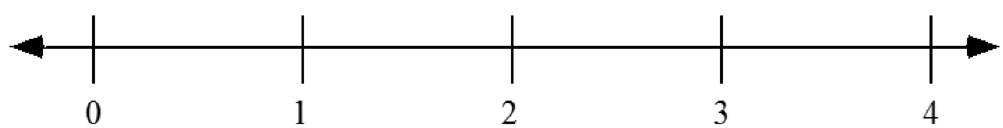
Exercise 1.6

1. Locate the indicated fractions on the number line. Remember to divide each whole into the appropriate number of equal parts as indicated by the denominator of the fraction.

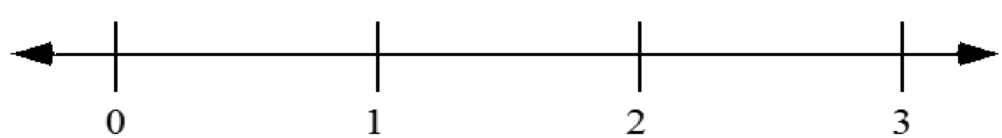
   a. $\frac{1}{2}, 2\frac{1}{2}, 3\frac{1}{2}, \frac{3}{2}$ (Hint: Divide each whole into 2 equal parts.)



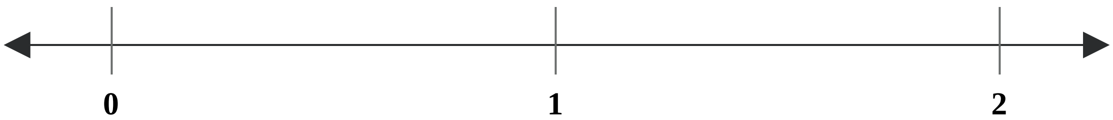
   b. $\frac{3}{5}, \frac{8}{5}, 2\frac{1}{5}, \frac{15}{5}$ (Hint: Divide each whole into 5 equal parts.)



   c. $\frac{3}{6}, \frac{5}{6}, \frac{9}{6}, \frac{12}{6}, 2\frac{1}{6}$



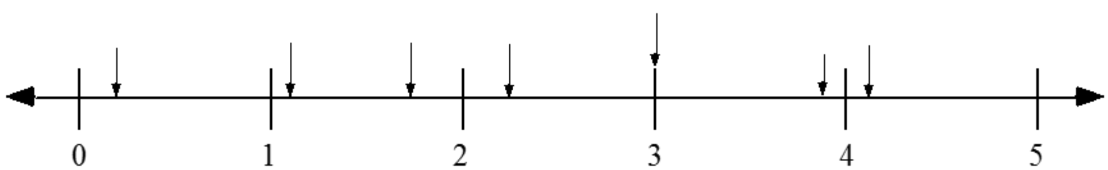
d. $\frac{1}{10}$, $\frac{5}{10}$, $\frac{11}{10}$, $\frac{13}{10}$



2. Draw a number line and label the ends 0 and 5. Show the approximate location of these fractions on the number line.

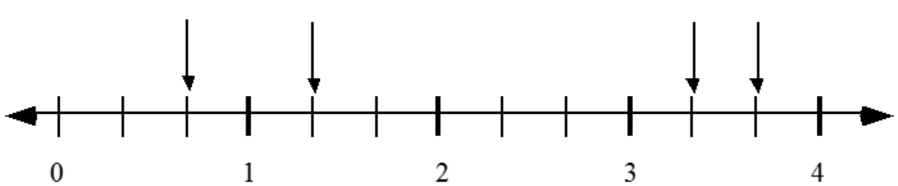
$4\frac{7}{8}$, $\frac{11}{16}$, $\frac{24}{6}$, $2\frac{2}{3}$, $1\frac{3}{4}$, $3\frac{1}{5}$

3. Write the correct fraction above each arrow. Select from the list below the number line.



$3\frac{9}{10}$, $1\frac{1}{10}$, $2\frac{3}{4}$, $\frac{1}{5}$, $4\frac{3}{4}$, $2\frac{1}{4}$, $\frac{3}{5}$, $\frac{3}{1}$, $1\frac{3}{4}$, $4\frac{1}{10}$

4. Write the correct fraction above each arrow.



Rounding a Fraction to the Nearest Whole Number

Introductory Video

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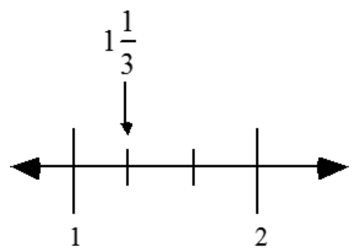
The rules for rounding are as follows:

✦ If the fraction is ½ **or more**, increase the whole number by one. If the fraction is **less than** ½, leave the whole number as it is.

Examples:

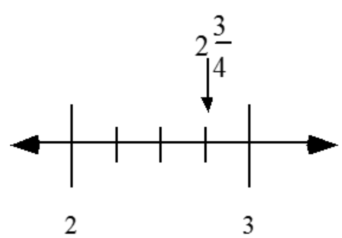
1. Round $1\frac{1}{3}$ to the nearest whole number.



$1\frac{1}{3}$ is closer to 1 than 2.

$1\frac{1}{3}$ rounds to 1.

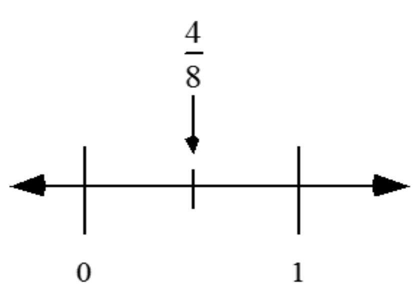
2. Round $2\frac{3}{4}$ to the nearest whole numbers



$2\frac{3}{4}$ is closer to 3 than 2.

$2\frac{3}{4}$ rounds to 3.

3. Round $\frac{4}{8}$ to the nearest whole number.



$\frac{4}{8}$ is exactly half way between 0 and 1. It is equal to $\frac{1}{2}$.

Thus we round $\frac{4}{8}$ to 1.

4. Round $\frac{1}{8}$ to the nearest whole number.

$\frac{1}{2} = \frac{4}{8}$ therefore, $\frac{1}{8}$ is less than $\frac{1}{2}$, so $\frac{1}{8}$ rounds to 0.

Exercise 1.7

1. Plot the following numbers on a number line first, then decide which whole number they are closest to.

   a. $2\frac{3}{8}$ rounds to ____

   b. $1\frac{2}{3}$ rounds to ____

2. Round these mixed numbers to the nearest whole number.

   a. $4\frac{1}{4} =$

   b. $3\frac{2}{3} =$

   c. $9\frac{3}{8} =$

   d. $5\frac{4}{5} =$

   e. $7\frac{7}{16} =$

   f. $8\frac{1}{2} =$

   g. $14\frac{5}{9} =$

   h. $9\frac{3}{4} =$

   i. $8\frac{5}{12} =$

   j. $12\frac{3}{7} =$

Using Rounding to Estimate Answers

Sometimes you only want to estimate an answer using fractions. That is, you only need a rough idea of the answer, not an exact figure. For example, you may be estimating the number of litres of water needed in a wash basin.You can use rounding off to **change fractions to whole numbers**, and then use the whole numbers to easily do the operations you need.

Examples:

1. Estimate the sum of $8\frac{1}{5} + 3\frac{4}{7}$.

   - $8\frac{1}{5}$ rounds to 8 because 1 (numerator) is **less than half** of 5 (denominator).
   - $3\frac{4}{7}$ rounds to 4 because 4 (numerator) is **more than half** of 7 (denominator).

   Therefore, the estimated sum is 8 + 4 = 12.

   Answer: $8\frac{1}{5} + 3\frac{4}{7} \cong 12$ (**Note:** $\cong$ means *approximately equal to*)

2. Estimate the product when $4\frac{1}{4}$ is multiplied by $3\frac{5}{6}$.

   - $4 \times 4 = 16$

   Answer: $4\frac{1}{4} \times 3\frac{5}{6} \cong 16$

Exercise 1.8

Estimate the following by rounding to the nearest whole number, then doing the required operation.

1. $3\frac{5}{8} + 2\frac{11}{16} \cong$
2. $22\frac{1}{4} + 5\frac{7}{9} \cong$
3. $42\frac{1}{2} - 28\frac{5}{8} \cong$
4. $18\frac{3}{5} - 4\frac{1}{10} \cong$
5. $3\frac{1}{4} \times 4\frac{1}{6} \cong$
6. $5\frac{3}{4} \times 2\frac{1}{2} \cong$

Factors

Common Error students will forget to include numbers when they don't use the U form see example below **Learner will see an intro video** https://youtu.be/XjHmByZB0wA

20

$1 \times 20 = 20$

$4 \times 5 = 20$ in this example a student has a better chance of missing a number1,20,4,5,2 miss 10

$2 \times 10 = 20$

Using the u shape below helps to not miss number. You can use this method with 2 or more numbers to find the greatest common factor.

20

1×20

2×10

4×5

Factors are any numbers multiplied together to give a product.

$$\underbrace{2 \times 2}_{\text{Factors}} = \underbrace{4}_{\text{Product}}$$

$2 \times 2 = 4$

$1 \times 4 = 4$

The **factors** of 4 are 1, 2, and 4.

1 is a **factor** of all numbers because of the **properties** of one.

Example: List the factors of 30.

Ask what two numbers (**factors**) can be multiplied together to make the answer (**product**) 30? Notice from the smallest to the largest forms a U shape.

30

$1 \times 30 = 30$

$2 \times 15 = 30$

$3 \times 10 = 30$

$5 \times 6 = 30$

Smallest to largest forms a U shape

The factors of 30 are 1, 2, 3, 5, 6, 10, 15, and 30.

Student practice:

Instructor led

For each number, find all the factors. Notice if you start with the smallest and go around to the largest, it forms a U shape.

1. 36

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2. 120

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Greatest Common Factor

The **greatest common factor (GCF)** is a number that divides evenly into two numbers, and it is the **largest** of all the factors that divides evenly into two numbers.

To find the greatest common factor, list **all** the factors of the two numbers. The largest number that is included in both is the greatest common factor

Example: Find the GCF of 18 and 24.

1. List all the factors of each one. What two numbers multiplied together make 18? Make 24?
   - Start with 1. Ask 1× what number makes 18? Then go to 2. Ask 2× what number makes 18? Then 3, and so on.

18	24
$1 \times 18 = 18$	$1 \times 24 = 24$
$2 \times 9 = 18$	$2 \times 12 = 24$
$3 \times 6 = 18$	$3 \times 8 = 24$
	$4 \times 6 = 24$

2. List the factors in order.

   18 = 1, 2, 3, 4, 6, 9, 18
   24 = 1, 2, 3, 4, 6, 8, 12, 24

3. Find the common factors of the two numbers.

   1, 2, 3, 6

4. The greatest (or biggest) of these common numbers is 6.
5. **The greatest common factor or GCF of 18 and 24 is 6.**

Student practice:

Instructor led

Find all the factors and circle the greatest common factor for the following:

1. 12 and 8

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2. 25 and 20

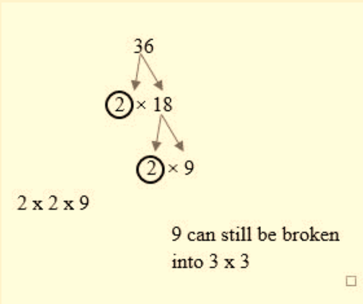
Exercise 1.9

Find all the factors of the following sets of numbers. Then list all the common factors for each set. Circle the greatest common factor.

1. 8 and 20
2. 16 and 24
3. 30 and 54
4. 27 and 72
5. 20 and 45
6. 30 and 50
7. 50 and 75
8. 56 and 84

Product of Prime Factors

Common Error not putting all of the factors in prime factorization
Example



Student will see intro video https://youtu.be/XGbOiYhHY2c

Prime numbers are numbers that have only two factors, the number 1 and itself.

- 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 87, 89, 93, 97
  Notice that all the prime factors except 2 are odd numbers, but not all odd numbers are prime.

Composite numbers are numbers that have more than two factors.

- 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21 …
  Notice that all even numbers except 2 are composite.

Writing Numbers as a Product of Prime Factors

Every composite number can be written as a product of prime factors.

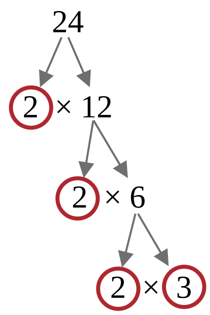
Using the division method, divide by the lowest prime number and continue until you end with 1.

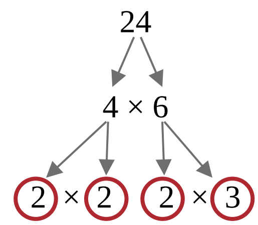
Example:

24
$24 \div 2 = 12$
$12 \div 2 = 6$
$6 \div 2 = 3$
$3 \div 3 = 1$

$2 \times 2 \times 2 \times 3 = 24$

Use the factor tree method to find all the factors of 24, then circle the prime factors.





Answer in repeated multiplication: $2 \times 2 \times 2 \times 3 = 24$

Answer in exponential form: $2^3 \times 3 = 24$

Student practice: Instructor led

Write the following factors as a product of prime factors. Use the division method and the factor tree.

1. 75

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2. 36

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https://youtu.be/XWq8bplP-_E

Exercise 1.10

Write the following numbers as a product of prime factors using either the division method or the factor tree method. Write them as repeated multiplication and in exponential form.

1. 12

2. 18

3. 36

4. 70

5. 63

6. 45

7. 81

8. 90

9. 100

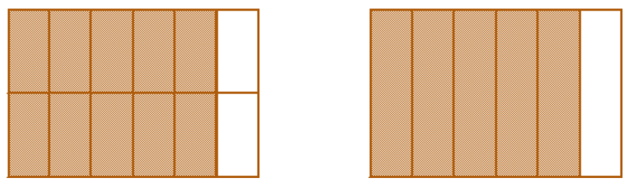
10. 700

Reducing Fractions

Common errors.

- Not reducing all the way $\frac{8 \div 2}{12 \div 2} = \frac{4}{6}$ can still reduce by dividing by 2 again.
- Not dividing the numerator and denominator by the same number $\frac{4 \div 2}{12 \div 3} = \frac{2}{4}$
- Prime factorization can be used to reduce fractions When the numerator and denominator are both in prime factorization numbers that are identical can be cancelled and changed to 1

Example $\frac{21}{28} = \frac{3 \bullet 7^{1}}{4 \bullet 7_{1}} = \frac{3}{4}$ student will see into video https://youtu.be/AtBUQH8Tkqc



$$\frac{10}{12} = \frac{5}{6}$$

To reduce fractions, two smaller parts are combined. Now the whole is made of 6 equal parts instead of 12.

Method 1

When reducing a fraction, always look for the greatest number that **divides evenly** into the numerator and denominator. This number is called the **greatest common factor (GCF)**.

Examples:

1. $\frac{21}{28} = \frac{21 \div 7}{28 \div 7} = \frac{3}{4}$ The GCF is 7.
2. $\frac{24}{36} = \frac{24 \div 12}{36 \div 12} = \frac{2}{3}$ The GCF is 12.

Hints to Help Reducing Fractions

- **A number can be divided evenly by 2** if the last digit is even; that is 0, 2, 4, 6, 8 (e.g., 24, 8, 66, 142, 30 can all be divided by 2).
- **A number can be divided evenly by 3** if the sum of the digits is a number that can be divided by 3 (e.g., 81, 8 + 1 = 9 and 9 divides by 3, so 81 divides by 3).

- **A number can be divided evenly by 5** if the last digit is 0 or 5 (e.g., 5, 10, 25, 60, 255 can all be divided by 5).
- **A number can be divided evenly by 10** if the last digit is 0 (e.g., 10, 200, 40, 560 can all be divided by 10).

All fractional answers should be written in lowest terms.

Method 2

Write fractions as a product of prime factors and then reduce. Cancel any number from the numerator that is identical to a number in the denominator.

Examples:

1. $\frac{21}{28} = \frac{3 \bullet \cancel{7}^1}{4 \bullet \cancel{7}_1} = \frac{3}{4}$

2. $\frac{24}{36} = \frac{\cancel{2}^1 \bullet \cancel{2}^1 \bullet 2 \bullet \cancel{3}^1}{\cancel{2}_1 \bullet \cancel{2}_1 \bullet \cancel{3}_1 \bullet 3} = \frac{2}{3}$

Student practice:

Instructor led

1. $\frac{48}{64}$

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2. $\frac{12}{42}$

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https://youtu.be/i3kL0DY51KE

Exercise 1.11

1. Reduce the following fractions to lowest terms.

a. $\frac{6}{9} =$ b. $\frac{9}{15} =$ c. $\frac{32}{36} =$

d. $\frac{8}{12} =$ e. $\frac{12}{18} =$ f. $\frac{40}{56} =$

g. $\frac{10}{24} =$ h. $\frac{21}{27} =$ i. $\frac{54}{60} =$

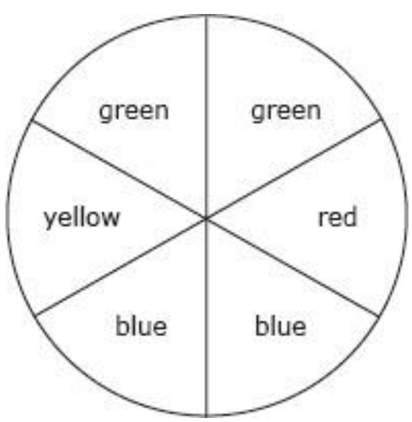
2. Change each of the following improper fractions to a whole number or a mixed number. Any fractional part **should be reduced** to its lowest term.

a. $\frac{8}{8} =$ b. $\frac{7}{5} =$ c. $\frac{12}{9} =$

d. $\frac{5}{2} =$ e. $\frac{15}{5} =$ f. $\frac{12}{3} =$

g. $\frac{6}{2} =$ h. $\frac{18}{3} =$ i. $\frac{16}{2} =$

3. What fraction of the circle is each colour?



a. yellow: ________________________________

b. green: ________________________________

c. green + red: ________________________________

d. green + blue: ________________________________

e. blue + yellow + green:________________________

f. red + blue + yellow + green: ________________

Unit 2: Addition and Subtraction of Fractions

Think about …

✦ You have been asked to provide your famous muffins for a potluck you are to attend with family. You have to double the recipe, which means doubling the ingredients. Have you ever found yourself trying to add up the amounts of ingredients for a recipe?

Teaching strategy. Have the learner rewrite the addition or subtraction statement with one denominator- example $\frac{5}{6}+\frac{3}{6}=\frac{5+3}{6}$

Common errors

- Adding the denominators together if the learner get used to writing the fraction over one denominator they won't make this mistake.
- Not reducing answers to lowest term. $\frac{5+3}{6}=\frac{8}{6}=1\frac{2}{6}$ $\frac{2}{6}$ can be reduce to $\frac{1}{3}$

Keywords

***Like* fractions**	Fractions that have the same denominator or common denominators
***Unlike* fractions**	Fractions that **do not** have the same denominator

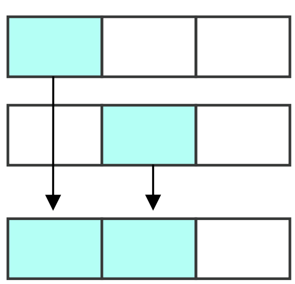
Addition of *Like* Fractions

***Like* fractions** have the same denominator or common denominator.

Fraction strips can be used to add fractions. To use fraction strips to add, find two strips that represent the given fractions. Where the first fraction ends, place the second strip. To find the sum, find a strip that is exactly the same length as both fractions or matches the sum.

Examples:

1. $\frac{1}{3}+\frac{1}{3}$



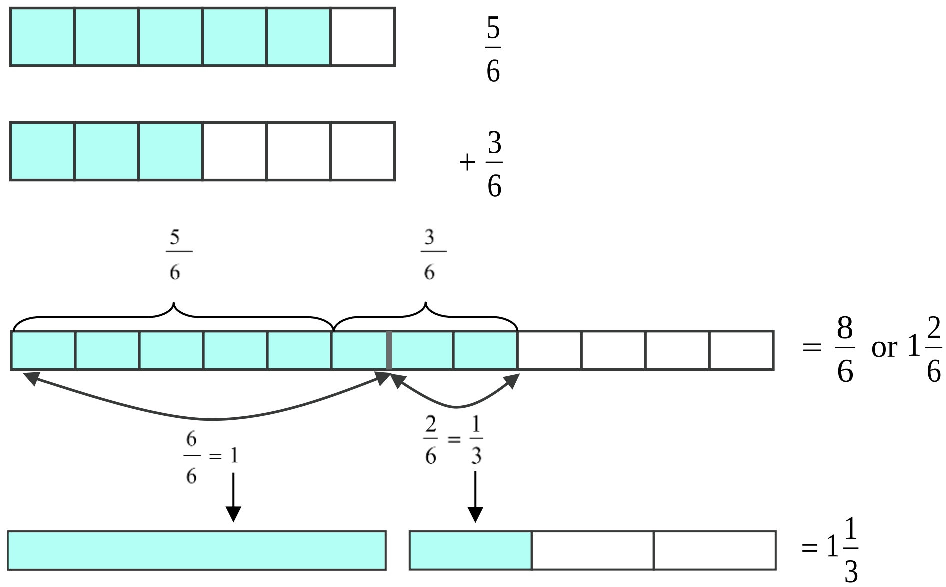
$\frac{1}{3}$

$+\frac{1}{3}$

$=\frac{2}{3}$

Thus, $\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$

2. $\frac{5}{6} + \frac{3}{6}$



Thus $\frac{5}{6} + \frac{3}{6} = \frac{8}{6} = 1\frac{2}{6} = 1\frac{1}{3}$

To add *like* fractions, we only need to add the numerators and reduce to lowest terms.

For mixed numbers, add the whole numbers, and then add the *like* fractions. The final answer should be reduced to lowest terms, if necessary.

Examples:

1.
$$\begin{array}{r} \frac{3}{7} \\[4pt] \frac{2}{7} \\[4pt] +\ \frac{1}{7} \\ \hline \end{array}$$

$$= \frac{3+2+1}{7} = \frac{6}{7}$$

2.
$$\begin{array}{r} 3\,\frac{4}{10} \\[4pt] \frac{2}{10} \\[4pt] +\ 2\,\frac{6}{10} \\ \hline \end{array}$$

$$= (3+2)\frac{4+2+6}{10} = 5\frac{12}{10} = 6\frac{2}{10} = 6\frac{1}{5}$$

Student practice:

Instructor led

1. $$\begin{array}{r} \dfrac{3}{15} \\ + \dfrac{7}{15} \\ \hline \end{array}$$

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https://youtu.be/EJjnEau6aeI

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Exercise 2.1

1. Use fraction strips or draw pictures to find each sum.

   a. $\frac{1}{5} + \frac{3}{5}$ b. $\frac{3}{4} + \frac{3}{4}$ c. $\frac{3}{8} + \frac{7}{8}$

2. Find the sums by adding the numerators of these like fractions. Reduce the answers to lowest terms.

   a. $$\begin{array}{r} \dfrac{2}{5} \\ + \dfrac{2}{5} \\ \hline \end{array}$$

   b. $$\begin{array}{r} \dfrac{3}{7} \\ + \dfrac{5}{7} \\ \hline \end{array}$$

   c. $$\begin{array}{r} \dfrac{16}{25} \\ + \dfrac{18}{25} \\ \hline \end{array}$$

d. $\dfrac{3}{6} + \dfrac{5}{6}$

e. $\dfrac{5}{16} + \dfrac{14}{16}$

f. $\dfrac{1}{4} + \dfrac{1}{4}$

g. $\dfrac{3}{8} + \dfrac{1}{8}$

h. $\dfrac{7}{12} + \dfrac{1}{12}$

i. $\dfrac{13}{32} + \dfrac{15}{32}$

j. $\dfrac{27}{64} + \dfrac{21}{64}$

k. $\dfrac{5}{6} + \dfrac{5}{6}$

l. $\dfrac{11}{16} + \dfrac{9}{16}$

Adding Whole Numbers, Fractions, and Mixed Numbers

Experience tells us that if we have 2½ dollars and we find 3 dollars, we now have 5½ dollars. To add any whole number to a fraction or mixed number, simply add the whole numbers and write the sum with the fraction.

Examples:

1. $2 + \frac{1}{3} = 2\frac{1}{3}$
2. $7 + 4\frac{1}{2} = 11\frac{1}{2}$
3. $4\frac{1}{3} + 5 = 9\frac{1}{3}$

Student Practice

Instructor led

$$\begin{array}{r} 2\frac{5}{13} \\ +\ 7\frac{6}{13} \\ \hline \end{array}$$

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Exercise 2.2

Solve the following equations.

1. $4 + \frac{2}{3}$
2. $\frac{1}{5} + 6$
3. $9 + \frac{3}{5}$
4. $9 + 8\frac{1}{7}$
5. $12\frac{5}{9} + 4$
6. $16\frac{1}{4} + 3$
7. $7\frac{1}{2} + 3$
8. $\frac{11}{12} + 4$
9. $5 + \frac{7}{9}$

10. $16\frac{2}{3}+12$

11. $\frac{1}{6}+6$

12. $3+16\frac{1}{3}$

13. $22\frac{7}{9}+4\frac{4}{9}$

14. $4\frac{1}{5}+7\frac{3}{5}$

15. $10\frac{2}{3}+8\frac{1}{3}$

16. Joe takes two ¼-hour coffee breaks and a 1 hour break for lunch. How much break time does he take in all?

Multiples

Learners can find multiples of a number by multiplying the number by 2, then 3 the 4 etc. or the learner can continue adding the number with itself as many times as needed

Common errors

- Finding a common multiple but not the lowest

Learners will see Intro Video to multiples https://youtu.be/rUrLuTMq-sw

A multiple of a number is the **product** or **answer** of that number multiplied by any whole number.

To find multiples of any number, just multiply that number by 1, 2, 3, 4, and on.

Example: Find multiples of 6.

Multiply 6 by 1, 2, 3, 4, and on.

$6 \times 1 = 6$
$6 \times 2 = 12$
$6 \times 3 = 18$
$6 \times 4 = 24$
$6 \times 5 = 30$

Answer: The multiples of 6 are: 6, 12, 18, 24, 30 ...

Least Common Multiples

There are two methods to finding the **least common multiple (LCM)** of numbers.

Method 1

List the multiples of each number.

Example: Find the common multiples of 6 and 8. Then find the least common multiple.

The multiples of 6 are: 6, 12, 18, 24, 30, 36, 42, 48, and on.

The multiples of 8 are: 8, 16, 24, 32, 40, 48, 56, 64, and on.

Answer: The common multiples of 6 and 8 are **24 and 48**.
The least common multiple is **24**.

This method works well if the numbers are small. For larger numbers, the next method will work better.

Method 2

Write each number as a product of prime factors. Then build the LCM from these.

Examples:

1. Find the lowest common multiple of 6 and 8.

6	**8**
2×3	$2 \times 2 \times 2$

To combine these numbers to find the lowest common multiple, we need to use the largest group of 2s, then the largest group of 3s, and continue the process. If we wanted to make the numbers the same, we would have to multiply the 6 by 2×2 and multiply the 8 by 3.

6	**8**
$2 \times 3 \times (2 \times 2) = 24$	$2 \times 2 \times 2 \times (3) = 24$

Answer: The lowest common multiple of 6 and 8 is **24**.

2. Find the lowest common multiple of 9 and 12.

9	**12**
3×3	$2 \times 2 \times 3$

The largest group of 2s is from the 12, and the largest group of 3s is from the 9.

Therefore $2 \times 2 \times 3 \times 3 = 36$ LCM

Answer: The lowest common multiple of 9 and 12 is **36**.

Student practice: Instructor led

1. Find the LCM of 30 and 25.

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2. Find the LCM of 15, 6, and 10.

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Exercise 2.3

Find the lowest common multiple for each set of numbers.

1. 8 and 10
2. 12 and 18
3. 8 and 20
4. 16 and 20
5. 14 and 35
6. 20 and 35
7. 2, 3, and 4
8. 4, 5, and 6
9. 6, 7, and 9
10. 10, 12, and 20

Adding *Unlike* Fractions

Unlike fractions **do not** have common denominators.

Before *unlike* fractions can be added (or subtracted), they must be changed to equivalent ***like* fractions** having a **common denominator**.

Our first problem then is to find a common denominator when adding *unlike* fractions.

- Find the sum: $\frac{2}{3} + \frac{3}{4}$

  The largest denominator of the two fractions is 4. List multiples of 4 until one is found that is exactly divisible by 3. (To find multiples of 4, just count by 4s).

  - Some multiples of 4 are 4, 8, 12, 16, 20, 24 ...

  Of the multiples listed, there are two that are exactly divisible by 4: 12 and 24. (There are more!) We are interested in **12** because it is the **lowest common multiple** of 3 and 4.

  Now, to add ⅔ and ¾, both must be changed to **equivalent fractions** with 12 as their denominator. This is called the **lowest common denominator (LCD)**.

$$\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12} \qquad \frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12}$$

Thus:

$$\begin{array}{r}
\frac{2}{3} = \frac{8}{12} \\
+\ \frac{3}{4} = \frac{9}{12} \\
= \frac{8+9}{12} = \frac{17}{12} = 1\frac{5}{12}
\end{array}$$

Step 1: Find the LCD.

Step 2: Change the *unlike* fractions to equivalent fractions based on the LCD.

Step 3: Add the numerators of the *like* fractions.

Step 4: Change all improper fractions to mixed numbers in lowest terms.

Student practice:

Instructor led

$\frac{4}{9} + \frac{11}{12}$

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Exercise 2.4

Add the fractions and reduce all answers to lowest terms, if necessary.

1. a. $\frac{4}{5} + \frac{7}{10}$ b. $\frac{6}{8} + \frac{7}{9}$ c. $\frac{7}{15} + \frac{5}{12}$

2. a. $\frac{3}{5} + \frac{8}{15}$ b. $\frac{3}{5} + \frac{5}{6}$ c. $\frac{2}{3} + \frac{7}{12}$

3. a. $\frac{1}{4} + \frac{5}{8} + \frac{3}{10}$ b. $\frac{5}{6} + \frac{2}{3} + \frac{7}{9}$ c. $\frac{3}{5} + \frac{7}{9}$

4. a. $\frac{3}{8} + \frac{7}{16}$ b. $\frac{1}{4} + \frac{1}{12}$ c. $\frac{3}{5} + \frac{1}{6}$

5. a. $\frac{11}{18} + \frac{7}{12}$ b. $\frac{3}{7} + \frac{2}{5}$ c. $\frac{3}{4} + \frac{5}{18}$

6. a. $\frac{5}{9} + \frac{5}{18}$ b. $\frac{1}{7} + \frac{5}{14}$ c. $\frac{1}{16} + \frac{5}{12}$

7. Andrew dug ⅔ of the depth required for a well on the first day. He dug ¼ more of the depth the next day. What fraction of the depth for the well had been dug? How much of the depth was left?

Adding Mixed Numbers

$6\frac{2}{5} + 4\frac{3}{4} =$

Estimate first. $6\frac{2}{5}$ rounds to 6, $4\frac{3}{4}$ rounds to 5, 6+5=30

$6\frac{2\times4}{5\times4} + 4\frac{3\times5}{4\times5} =$

Find the lowest common denominator

$6\frac{8}{20} + 4\frac{15}{20} =$

Add whole numbers then add fractions

$6+4 \quad \frac{8+15}{20} = 10\frac{23}{20} = 11\frac{3}{20}$

Reduce to lowest terms

Student Practice

Instructor led

1. $2\frac{1}{6} + 4\frac{2}{3}$

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2. $3\frac{1}{12} + 11\frac{2}{5} + 4\frac{3}{15}$

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Exercise 2.5

Estimate, by rounding to the nearest whole number then add the following mixed numbers. Reduce to lowest terms.

1. $2\frac{3}{8}+1\frac{1}{4}$

2. $7+\frac{5}{16}$

3. $1\frac{2}{5}+9\frac{7}{10}$

4. $6\frac{1}{4}+3\frac{3}{8}$

5. $4\frac{9}{16}+5\frac{1}{2}$

6. $5\frac{1}{4}+8\frac{5}{6}$

7. $23\frac{9}{10}+12\frac{3}{8}$

8. $12\frac{2}{3}+2\frac{7}{12}$

9. $4\frac{3}{5}+7\frac{3}{4}$

10. $17\frac{1}{4}+9\frac{2}{3}$

11. $5\frac{4}{5}+8\frac{7}{10}+9\frac{3}{4}$

12. $6\frac{1}{4}+3\frac{3}{8}+1\frac{7}{16}$

13. $14\frac{2}{3}+8\frac{3}{5}+15\frac{1}{4}$

14. $12\frac{1}{2}+23\frac{5}{6}+18\frac{3}{4}$

15. $28\frac{11}{12}+15\frac{5}{9}+17\frac{5}{6}$

16. $8\frac{2}{5}+24\frac{2}{3}+9\frac{2}{6}$

Subtracting *Like* Fractions

To subtract *like* fractions, simply subtract the numerators and place the result over the common denominator. The final answer should be reduced to lowest terms, if necessary.

Example: $\frac{13}{15} - \frac{9}{15} = \frac{13-9}{15} = \frac{4}{15}$

Student practice:

Instructor led

$\frac{8}{18} - \frac{5}{18} =$

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Exercise 2.6

Subtract the fractions and reduce the answers to lowest terms, if necessary.

1. $\frac{3}{4} - \frac{1}{4}$
2. $\frac{5}{6} - \frac{1}{6}$
3. $\frac{3}{8} - \frac{3}{8}$
4. $\frac{5}{8} - \frac{3}{8}$
5. $\frac{3}{5} - \frac{1}{5}$
6. $\frac{7}{8} - \frac{3}{8}$
7. $\frac{7}{9} - \frac{4}{9}$
8. $\frac{11}{12} - \frac{5}{12}$
9. $\frac{7}{10} - \frac{3}{10}$

Subtracting *Unlike* Fractions

To subtract *unlike* fractions, a common denominator must be found. Before *unlike* fractions can be subtracted, they must be changed to equivalent *like* fractions having a common denominator. (This step is the same as for adding *unlike* fractions.)

Step 1: Find the LCD

Step 2: Change the *unlike* fractions to *like* fractions based on the LCD.

Step 3: Subtract the numerators of the *like* fractions.

Step 4: Reduce to lowest terms

Example:

$$\begin{aligned} &\frac{5}{6} = \frac{20}{24} \\ -&\frac{3}{8} = \frac{9}{24} \\ &\quad = \frac{20-9}{24} = \frac{11}{24} \end{aligned}$$

Student practice:

Instructor led

1. $\frac{4}{3} - \frac{1}{5}$

2. $\frac{7}{10} - \frac{5}{8}$

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Exercise 2.7

Subtract the fractions and reduce the answers to lowest terms, if necessary.

1. a. $\begin{array}{r} \frac{5}{6} \\ -\frac{2}{3} \\ \hline \end{array}$ b. $\begin{array}{r} \frac{3}{4} \\ -\frac{1}{2} \\ \hline \end{array}$ c. $\begin{array}{r} \frac{1}{4} \\ -\frac{1}{12} \\ \hline \end{array}$

d. $\begin{array}{r} \frac{7}{10} \\ -\ \frac{3}{5} \\ \hline \end{array}$ e. $\begin{array}{r} \frac{3}{4} \\ -\ \frac{5}{8} \\ \hline \end{array}$ f. $\begin{array}{r} \frac{15}{16} \\ -\ \frac{5}{8} \\ \hline \end{array}$

2. a. $\frac{7}{16} - \frac{1}{4}$ b. $\frac{7}{10} - \frac{1}{2}$ c. $\frac{2}{5} - \frac{1}{10}$

d. $\frac{7}{8} - \frac{3}{5}$ e. $\frac{9}{12} - \frac{1}{2}$ f. $\frac{4}{5} - \frac{1}{20}$

g. $\frac{11}{15} - \frac{1}{2}$ h. $\frac{11}{16} - \frac{1}{4}$ i. $\frac{3}{8} - \frac{1}{3}$

j. $\frac{7}{16} - \frac{3}{8}$ k. $\frac{5}{8} - \frac{1}{6}$ l. $\frac{13}{15} - \frac{7}{10}$

Subtracting a Fraction from a Whole Number

A fraction is needed to subtract a fraction. Borrow and make the fraction with the same denominator that is being subtracted

Example $3-1\frac{2}{5}$ since the denominator is 5 as we borrow one from the 3 it becomes $2\frac{5}{5}$

Now there is a fraction to subtract a fraction $2\frac{5}{5}-1\frac{2}{5}$ now subtract 2-1 =1 $\frac{5}{5}-\frac{2}{5}=\frac{5-2}{5}=1\frac{3}{5}$

Example:

$2-\frac{2}{3}=$ **Step 1:** Borrow from the whole number.

$1\frac{3}{3}-\frac{2}{3}=$ **Step 2:** Rewrite the equation using *like* fractions.

$1\frac{1}{3}$ **Step 3:** Subtract the fractions

$$\begin{array}{r} 10\frac{1}{3} \\ -\ 6 \\ \hline 4\frac{1}{3} \end{array}$$

Watch it!! We don't have to borrow here because nothing is being subtracted from the fraction ⅓. Just subtract the whole numbers.

Student practice:

Instructor led

$4-2\frac{2}{7}$

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https://youtu.be/6Dm5dPvlvw4

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Exercise 2.8

Solve the equations.

1. $2 - \frac{3}{4}$
2. $4\frac{1}{3} - 2$
3. $2\frac{1}{2} - 1$
4. $3 - 1\frac{1}{2}$
5. $4 - 1\frac{1}{3}$
6. $4 - 2\frac{1}{2}$
7. $3 - 2\frac{7}{8}$
8. $9 - 3\frac{1}{5}$
9. $5 - 2\frac{1}{4}$
10. $7 - 2\frac{5}{6}$
11. $4 - 2\frac{2}{5}$
12. $1 - \frac{1}{3}$

Subtracting Mixed Numbers

Common Error: When borrowing learners forget to add the original numerator to the new one. Example $3\frac{1}{4} = 2\frac{4}{4}$ and forget to add the $\frac{1}{4}$. The best way to help learners is to teach when you borrow 1 add the denominator to the numerator $3\frac{1}{4} = 2\frac{4+1}{4} = 2\frac{5}{4}$.

Example $5\frac{1}{4} - 2\frac{3}{4}$ since the this number has a lower fraction borrow $5\frac{1}{4} = 4 + \frac{4+1}{4} = 4\frac{5}{4}$

$4\frac{5}{4} - 2\frac{3}{4}$ Now subtraction is possible $4\frac{5}{4} - 2\frac{3}{4} = 4 - 2 then \frac{5}{4} - \frac{3}{4} = 2\frac{2}{4} = 2\frac{1}{2}$

Learner will see intro video https://youtu.be/_btQus9HV_I

Step 1: Find the LCD.

Step 2: Change the *unlike* fractions to *like* fractions based on the LCD.

Step 3: Borrow if necessary.

Step 4: Subtract the whole numbers.

Step 5: Subtract the numerators of the *like* fractions.

Step 6: Reduce the answer to lowest terms.

Examples:

1.
$$\begin{array}{rcrcr} 6\frac{2}{5} & \rightarrow & 6\frac{8}{20} & \rightarrow & 5\frac{28}{20} \\ -\,4\frac{3}{4} & \rightarrow & 4\frac{15}{20} & \rightarrow & -\,4\frac{15}{20} \\ \hline & & & & 1\frac{13}{20} \end{array}$$

← Change to a common denominator.
Borrow.

2. $4\frac{2}{3} - 3\frac{3}{4}$

$$\begin{array}{rcrcr} 4\frac{2}{3} & \rightarrow & 4\frac{8}{12} & \rightarrow & 3\frac{20}{12} \\ -\,3\frac{3}{4} & \rightarrow & 3\frac{9}{12} & \rightarrow & -\,3\frac{9}{12} \\ \hline & & & & \frac{11}{12} \end{array}$$

Student practice:

Instructor led

$$17\frac{4}{9} - 12\frac{2}{3}$$

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Exercise 2.9

Solve the equations.

1. $2\frac{7}{8} - 1\frac{3}{8}$
2. $6\frac{1}{3} - 2\frac{2}{3}$
3. $8\frac{1}{2} - 2\frac{3}{10}$
4. $9\frac{1}{2} - 3\frac{3}{4}$
5. $14\frac{1}{3} - 11\frac{5}{6}$
6. $16\frac{1}{4} - 12\frac{3}{8}$
7. $13\frac{1}{6} - 2\frac{7}{8}$
8. $9\frac{3}{4} - 3\frac{5}{6}$
9. $15\frac{3}{4} - 12\frac{4}{7}$

10. $5\frac{3}{8} - 3\frac{3}{4}$

11. $4\frac{5}{8} - 2\frac{5}{6}$

12. $21\frac{1}{4} - 9\frac{3}{5}$

13. $11\frac{3}{4} - 5\frac{2}{7}$

14. $6\frac{4}{9} - 2\frac{1}{6}$

15. $13\frac{1}{2} - 1\frac{3}{9}$

16. $7\frac{7}{8} - 2\frac{3}{10}$

17. $23\frac{1}{4} - 12\frac{7}{9}$

18. $5\frac{1}{5} - 1\frac{4}{6}$

19. $2\frac{5}{6} - \frac{3}{4}$

20. $17\frac{3}{4} - 11\frac{2}{3}$

21. $2\frac{3}{4} - 1\frac{10}{12}$

22. $17\frac{1}{5} - 2\frac{6}{15}$

23. $14\frac{3}{8} - 2\frac{4}{5}$

24. $21\frac{3}{8} - 18\frac{6}{10}$

Unit 3: Multiplication and Division of Fractions

Keywords

Complex fraction	A fraction formed of two fractional expressions, one on top of the other.
Reciprocal	The number by which a given number must be multiplied to get a result of one. For example, the *reciprocal* of ½ is 2 ($\frac{1}{2}$ and $\frac{2}{1}$).

Multiplying Fractions

When multiplying fractions really just multiplying the numerators together and multiplying the denominators together. Learner sees intro videos https://youtu.be/zQDNnDxsxi8 and https://youtu.be/ZAaDRQix7Ss

Must have proper or improper fractions to multiply or divide.

Common errors not reducing to lowest terms. Have learners reduce any number from the top with any number from the bottom. This will reduce the size of the numbers they are multiplying and if they reduce all first they won't need to reduce at the end.

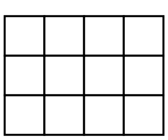
Example $\frac{5}{9}\times\frac{3}{10}=\frac{\cancel{5}^{1}}{9}\times\frac{3}{\cancel{10}_{2}}=\frac{\cancel{5}^{1}}{\cancel{9}_{3}}\times\frac{\cancel{3}^{1}}{\cancel{10}_{2}}=\frac{1\times1}{3\times2}=\frac{1}{6}$ Instead of $\frac{5}{9}\times\frac{3}{10}=\frac{15}{90}=\frac{15\div15}{90\div15}=\frac{1}{6}$

To multiply fractions, there is no need to convert denominators to *like* terms. Multiply numerator to numerator and denominator to denominator. Always write the product in simplest terms and, if an improper fraction results, change it to a mixed number or whole number.

Cancellation can be used before multiplying terms to simplify the multiplication process. To use cancellation, **divide one numerator *and* one denominator** by the greatest common factor (GCF). Study Example 4 below closely:

Examples:

1. A rectangle is 4 squares long and 3 squares wide.

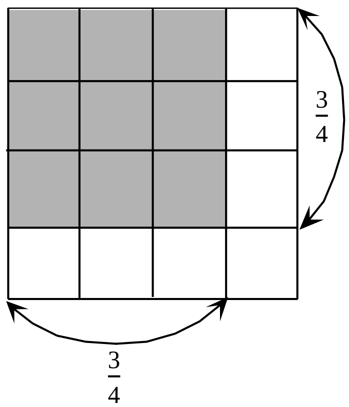


3

4

The total number of squares is $3 \times 4 = 12$.

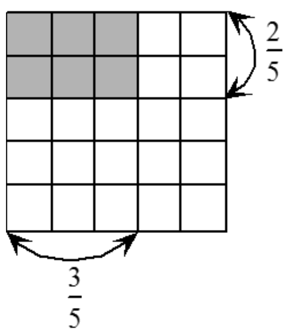
2. Multiply $\frac{3}{4}$ of a square by $\frac{3}{4}$ of a square.



The total number of shaded squares is 9 out of 16 **or**

$$\frac{3}{4} \times \frac{3}{4} = \frac{3 \times 3}{4 \times 4} = \frac{9}{16}$$

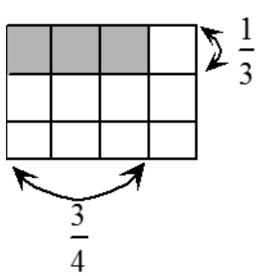
3. Multiply $\frac{2}{5}$ of a square by $\frac{3}{5}$ of a square.



The total number of shaded squares is 6 out of 25 **or**

$$\frac{2}{5} \times \frac{3}{5} = \frac{2 \times 3}{5 \times 5} = \frac{6}{25}$$

4. Multiply $\frac{3}{4}$ of a square by $\frac{1}{3}$ of a square.



The total number of shaded squares is 3 out of 12 **or**

$$\frac{3}{4} \times \frac{1}{3} = \frac{3 \times 1}{4 \times 3} = \frac{3}{12} = \frac{1}{4}$$

In this question, we can simplify first by using cancellation:

$$\frac{{}^{1}\cancel{3}}{4} \times \frac{1}{\cancel{3}_{1}} = \frac{1 \times 1}{4 \times 1} = \frac{1}{4}$$

To multiply any combination involving proper fractions, mixed numbers, or whole numbers:

Step 1: Change all whole numbers and mixed numbers to fraction form.

Step 2: Reduce (using cancellation) by dividing out common factors.

Step 3: Multiply the reduced fractions.

Step 4: Change any improper fractions to mixed numbers or whole numbers.

Examples:

1. $\frac{4}{6} \times \frac{3}{5} \times \frac{10}{12} = \frac{\cancel{4}^{1}}{\cancel{6}_{21}} \times \frac{\cancel{3}^{1}}{\cancel{5}_{1}} \times \frac{\cancel{10}^{21}}{\cancel{12}_{3}}$

$= \frac{1 \times 1 \times 1}{1 \times 1 \times 3}$

$= \frac{1}{3}$

2. $5 \times 1\frac{2}{5} = \frac{5}{1} \times \frac{7}{5}$

$= \frac{\cancel{5}^{1}}{1} \times \frac{7}{\cancel{5}_{1}}$

$= \frac{1 \times 7}{1 \times 1}$

$= \frac{7}{1} = 7$

Student practice:

Instructor led

$\frac{5}{6} \times \frac{2}{3}$

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Exercise 3.1

Try the following problems. Simplify before multiplying whenever possible.

1. $\frac{2}{3} \times \frac{3}{4}$

2. $\frac{1}{3} \times \frac{4}{9}$

3. $\frac{5}{9} \times \frac{3}{5}$

4. $\frac{7}{8} \times \frac{2}{3}$

5. $\frac{7}{16} \times \frac{2}{21}$

6. $\frac{9}{12} \times \frac{7}{8}$

7. $\frac{5}{9} \times \frac{7}{4}$

8. $\frac{9}{14} \times \frac{49}{15}$

9. $\frac{18}{77} \times \frac{11}{30}$

10. $\frac{14}{25} \times \frac{15}{28}$

11. $\frac{6}{7} \times \frac{35}{24}$

12. $\frac{15}{8} \times \frac{4}{25}$

13. $\frac{3}{5} \times \frac{9}{11} \times \frac{10}{27}$

14. $\frac{7}{10} \times \frac{25}{49} \times \frac{7}{5}$

15. $\frac{2}{3} \times \frac{4}{5} \times \frac{3}{7}$

16. $\frac{1}{2} \times \frac{3}{5} \times \frac{7}{3}$

17. $\frac{2}{5} \times \frac{9}{16} \times \frac{20}{24} \times \frac{10}{3}$

18. $\frac{1}{3} \times \frac{6}{7} \times \frac{21}{24} \times \frac{8}{9}$

19. $\frac{4}{11} \times \frac{22}{25} \times \frac{20}{12} \times \frac{5}{6} \times \frac{9}{8}$

20. $\frac{2}{3} \times \frac{6}{7} \times \frac{21}{24} \times \frac{3}{4} \times \frac{8}{9}$

Multiplying Mixed Numbers

When multiplying or divide mixed fractions, they must be converted to improper fractions. **Then follow the same procedure as with proper fractions**

Student Practice

Instructor led

$1\frac{3}{4} \times 7\frac{1}{5}$

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Exercise 3.2

Solve the following equations.

1. $8\frac{1}{3} \times 9$

2. $7\frac{1}{2} \times 6\frac{2}{3}$

3. $3\frac{3}{4} \times 3\frac{3}{5}$

4. $\frac{2}{5} \times 1\frac{3}{4}$

5. $3\frac{1}{3} \times 6$

6. $2\frac{4}{5} \times 1\frac{2}{3}$

7. $4\frac{1}{2} \times 2\frac{2}{3}$

8. $6\frac{2}{5} \times \frac{3}{8}$

9. $5\frac{1}{4} \times 1\frac{3}{7}$

10. $4\frac{2}{3} \times 5\frac{1}{4}$

11. Joe walks 6 blocks to school. He averages ¾ minutes to walk each block.

    a. How long does he take to walk to school?

    b. How long does he spend each week walking to and from school?

12. To finish a book report, Tony had to read 18½ pages. He read ½ of these on Sunday night. How many pages does he have left to read for Monday morning?

13. A cookie recipe calls for 2½ cups of flour, 1¾ cups of sugar, and ¼ cup of baking powder. Debra needs to make 2½ times the recipe. How much of each ingredient does she need?

Reciprocals

A reciprocal is the inverse of a fraction, therefore the reciprocal of $\frac{1}{2}$ is $\frac{2}{1}$

To find a reciprocal of a mixed fraction (change it to an improper fractions first)

Common error is to find the reciprocal of the fractional part only $1\frac{2}{3}$ reciprocal $1\frac{3}{2}$

Learners see intro video https://youtu.be/4lkq3DgvmJo

If the product of two fractions is 1, then the fractions are **reciprocals** of each other. An understanding of reciprocals is required when dividing fractions.

Examples:

1. $\frac{3}{4}$ and $\frac{4}{3}$ because $\frac{3}{4} \times \frac{4}{3} = \frac{12}{12} = 1$

2. $\frac{2}{5}$ and $\frac{5}{2}$ because $\frac{2}{5} \times \frac{5}{2} = \frac{10}{10} = 1$

3. 10 and $\frac{1}{10}$ because $10 \times \frac{1}{10} = \frac{10}{1} \times \frac{1}{10} = \frac{10}{10} = 1$

4. $1\frac{2}{3}$ and $\frac{3}{5}$ because $1\frac{2}{3} \times \frac{3}{5} = \frac{5}{3} \times \frac{3}{5} = \frac{15}{15} = 1$

Student practice: Instructor led

$2\frac{4}{5}$

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Exercise 3.3

Find the reciprocal of each of the following.

1. $\frac{1}{9}$
2. $\frac{6}{17}$
3. $\frac{12}{11}$
4. $4\frac{2}{5}$
5. 14
6. $\frac{10}{9}$
7. $3\frac{4}{15}$
8. $2\frac{5}{26}$
9. $\frac{40}{3}$
10. $\frac{5}{22}$
11. 2
12. $\frac{5}{4}$
13. $\frac{50}{3}$
14. $\frac{100}{61}$
15. $\frac{9}{20}$
16. 1

Dividing Fractions

> We never divide fractions, we multiply by the reciprocal.
> Common errors learners may change both fractions to their reciprocals and then multiply or they may change the first fraction to the reciprocal first and then multiply.
> Teach as they change the division sign to times then change the fraction behind to its reciprocal. Learner will see intro video **https://youtu.be/4lkq3DgvmJo**

Bob has four plots of land. The plots of land are shown below.

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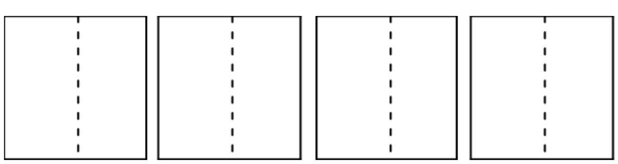
How many separate pieces will he have if he divides each of the four plots into ½ plots? Into ⅓ plots?

To answer these questions, think of a diagram.

½ plots:

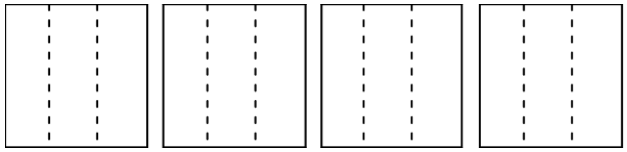


or eight ½ plots

You can interpret division of fractions using multiplication.

$$4 \div \frac{1}{2} = 8 \quad \textbf{or} \quad 4 \times \frac{2}{1} = 8$$

If Bob was to divide his 4 plots by ⅓:

⅓ plots:  or twelve ⅓ plots

$$4 \div \frac{1}{3} = 12 \quad \textbf{or} \quad 4 \times \frac{3}{1} = 12$$

To **divide** any number by a fraction, **multiply** the number by the **reciprocal of the fraction**. Therefore, a division problem is changed to a multiplication problem.

The following example will illustrate the statement above:

$$\frac{2}{3} \div \frac{7}{8} = \frac{2}{3} \times \frac{8}{7} = \frac{2 \times 8}{3 \times 7} = \frac{16}{21}$$

Steps for dividing fractions:

1. Change any whole or mixed numbers to improper fractions.
2. Multiply the first fraction by the reciprocal of the second fraction.
3. Proceed with the steps for multiplication.

Examples:

1. $3 \div \frac{2}{3} = \frac{3}{1} \div \frac{2}{3}$

$= \frac{3}{1} \times \frac{3}{2}$

$= \frac{3 \times 3}{1 \times 2} = \frac{9}{2} = 4\frac{1}{2}$

2. $3\frac{1}{2} \div \frac{3}{4} = \frac{7}{2} \div \frac{3}{4}$

$= \frac{7}{2} \times \frac{4}{3}$

$= \frac{7}{\cancel{2}_1} \times \frac{\cancel{4}^2}{3} = \frac{7 \times 2}{1 \times 3} = \frac{14}{3} = 4\frac{2}{3}$

3. $3\frac{3}{4} \div 1\frac{1}{6} = \frac{15}{4} \div \frac{7}{6}$

$= \frac{15}{4} \times \frac{6}{7}$

$= \frac{15}{\cancel{4}_2} \times \frac{\cancel{6}^3}{7} = \frac{15 \times 3}{2 \times 7} = \frac{45}{14} = 3\frac{3}{14}$

Student practice:

Instructor led

1. $\frac{3}{5} \div \frac{1}{2}$

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Exercise 3.4

Solve the equations.

1. $\frac{1}{8} \div \frac{1}{8}$
2. $\frac{2}{3} \div \frac{4}{9}$
3. $\frac{11}{12} \div \frac{3}{4}$
4. $22 \div \frac{1}{2}$
5. $15 \div \frac{3}{4}$
6. $2\frac{3}{8} \div 19$
7. $6\frac{3}{4} \div 9$
8. $\frac{2}{5} \div 1\frac{3}{5}$
9. $8\frac{1}{4} \div \frac{11}{20}$
10. $\frac{7}{8} \div 1\frac{5}{16}$
11. $5\frac{2}{9} \div \frac{1}{27}$
12. $1\frac{1}{2} \div \frac{3}{4}$

13. $\frac{3}{8} \div \frac{7}{8}$

14. $6 \div 7\frac{1}{2}$

15. $8\frac{1}{2} \div 6$

16. Cecil had $1\frac{2}{3}$ pizzas. He wanted each person to have $\frac{1}{3}$ of a pizza. How many friends could he share with?

17. Four people are running an 800-metre relay race. The track is 150 metres around.

    a. Each person runs 200 metres. How many times will each person run around the track?

    b. The average time to complete one lap of the track is $20\frac{1}{2}$ seconds. How long will each person take to run 200 metres?

    c. What is the total time the team needs to complete the relay?

Simplifying Complex Fractions

$\dfrac{\frac{2}{3}}{\frac{3}{4}}$, $\dfrac{2\frac{1}{2}}{4}$, $\dfrac{1\frac{3}{4}}{2\frac{3}{5}}$, and $\dfrac{5}{1\frac{1}{2}}$ are examples of **complex fractions**.

Note that the numerator and/or denominator may themselves be fractions.

To *simplify a complex fraction* means to divide the numerator by the denominator.

Examples:

1. $\dfrac{\frac{2}{3}}{\frac{3}{4}} = \dfrac{2}{3} \div \dfrac{3}{4}$ (Remember, the fraction bar means *divide*.)

$$= \frac{2}{3} \times \frac{4}{3} = \frac{2 \times 4}{3 \times 3} = \frac{8}{9}$$

2. $\dfrac{2\frac{1}{2}}{4} = 2\dfrac{1}{2} \div \dfrac{4}{1}$ (Recall that $4 = \dfrac{4}{1}$.)

$$= \frac{5}{2} \times \frac{1}{4} = \frac{5 \times 1}{2 \times 4} = \frac{5}{8}$$

3. $\dfrac{1\frac{3}{4}}{2\frac{3}{5}} = 1\dfrac{3}{4} \div 2\dfrac{3}{5} = \dfrac{7}{4} \div \dfrac{13}{5}$

$$= \frac{7}{4} \times \frac{5}{13} = \frac{35}{52}$$

4. $\dfrac{5}{1\frac{1}{2}} = \dfrac{5}{1} \div \dfrac{3}{2}$

$$= \frac{5}{1} \times \frac{2}{3} = \frac{10}{3} = 3\frac{1}{3}$$

Student Practice

$2\dfrac{1}{4} \div 1\dfrac{3}{4}$

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Exercise 3.5

Solve the following equations. Make sure that your answers are in simplest form.

1. $\dfrac{\frac{1}{2}}{6}$

2. $\dfrac{16}{\frac{8}{3}}$

3. $\dfrac{\frac{5}{8}}{20}$

4. $\dfrac{\frac{4}{5}}{\frac{4}{15}}$

5. $\dfrac{\frac{5}{6}}{\frac{25}{24}}$

6. $\dfrac{\frac{28}{15}}{\frac{21}{5}}$

7. $\dfrac{\frac{4}{7}}{8}$

8. $\dfrac{\frac{9}{10}}{\frac{3}{5}}$

9. $\dfrac{12}{\frac{3}{4}}$

10. $\dfrac{\frac{8}{3}}{16}$

11. $\dfrac{20}{\frac{5}{8}}$

12. $\dfrac{\frac{4}{15}}{\frac{4}{5}}$

Extra Exercise 3.6

Solve the following equations. Make sure that your answers are in simplest form.

1. $7\frac{1}{4} \div 2\frac{5}{12}$

2. $10 \div \frac{5}{10}$

3. $5\frac{1}{2} \div \frac{1}{3}$

4. $\frac{7}{16} \div 7$

5. $\frac{1}{4} \div 1\frac{1}{2}$

6. $3\frac{1}{2} \div 5\frac{5}{6}$

7. $1\frac{1}{3} \div 2\frac{1}{2}$

8. $\frac{7}{9} \div 2\frac{1}{3}$

9. $\frac{3}{10} \div 2\frac{1}{2}$

10. $4\frac{1}{5} \div \frac{7}{8}$

11. $2\frac{2}{3} \div 4$

12. $4\frac{3}{8} \div 2\frac{1}{12}$

13. $\dfrac{1\frac{7}{8}}{6\frac{1}{4}}$

14. $\dfrac{8\frac{2}{5}}{3\frac{1}{2}}$

Unit 4: Fraction Word Problems

Using Fractions in Word Problems

Examples:

1. There are 9 units of equal length in a math course. Tom has finished 2 units. What fraction of the course has he finished? What fraction of the course is still unfinished?

   Solution:

   Total units in the course = 9

   Units finished = 2

   Fraction of the course finished is $\frac{2}{9}$

   Fraction of the course unfinished is $\frac{7}{9}$

2. There are 20 weeks in a regular NorQuest College term. Jean has been at NorQuest College for 3 weeks. What fraction of a term does she have yet to complete?

   Solution:

   Total weeks in the term = 20

   Weeks to be completed = 20 – 3 = 17

   Fraction of the term left to be completed is $\frac{17}{20}$

Strategies for Problem Solving

Stuck? Frustrated? Don't know what to try next?

Pick one of these wonderful, "guaranteed to give you some sort of start" strategies.

Try

- ✓ searching for some sort of pattern
- ✓ drawing a diagram
- ✓ organizing your information into lists, tables, or charts
- ✓ making a guess and checking it out
- ✓ breaking the problem up into a few simpler problems
- ✓ working backwards
- ✓ eliminating some of the possibilities
- ✓ using logic or reasoning, think about the problem
- ✓ writing an arithmetic expression that describes the problem

And if none of these work, take a break, have a coffee, let the old subconscious play around with it a bit, then sit down and **start again**.

Exercise 4.1

Read each problem and write the fraction described.

1. Joel ploughed 32 rows for a garden. He planted 13 rows of corn. What fraction of the garden has he planted in corn?

2. Lin attends kindergarten 5 days a week. She missed 1 day this week. What fraction of the school week did she miss? What fraction of the week did she attend school?

3. On a baseball team, 4 of the starting 9 players have a beard. What fraction of the starters has a beard? What fraction does not have a beard?

4. The distance from Ahmed's house to school is 4 kilometres. He drives 3 kilometres and walks the rest of the way. What part of the distance does he drive? What part of the distance does he walk?

5. Of the 6 people in Mary’s family, she is the only one who likes to play tennis. What part of Mary’s family likes to play tennis? What part of the family does not like to play tennis?

6. Lucia bought a 250 mL cold drink and spilled 119 mL when she opened it. What part of the cold drink did she spill? What part did she have left?

7. Sue bought a dozen eggs at the grocery store. When she got home, she noticed that 5 of the eggs were broken. What part of the 12 eggs was broken? What part was not broken?

8. The City of Edmonton pothole brigade has 24 potholes to fill in 2 days. The first day, they filled 5 potholes. What fractional part of the potholes do they have yet to fill?

9. A new house has a kitchen, living room, three bedrooms, and two bathrooms.
    a. What fractional part of the rooms are bathrooms?
    b. What part of all the rooms does the kitchen represent?
    c. What part of the rooms are bedrooms?

10. A 400-page book has 21 pages with charts and graphs, and 38 pages with colour illustrations. The rest of the pages contain only printed material. What fraction of the book contains only printed material?

11. A math class is made up of 12 students between 28 and 35 years of age, 5 students between 19 and 27 years of age, and 2 students between 36 and 40 years of age. Eight of the students are female.
    a. What part of the class is female?
    b. What part of the class is between 19 and 35 years of age?
    c. What part of the class do the male students represent?

12. A traveller set out on a 210-kilometre journey by taking a bus for the first 100 kilometres. He then rented a car and drove 90 kilometres before it broke down. He completed the rest of the journey on foot.
    a. What part of the journey was completed by bus?
    b. What part was completed by car?
    c. What part was completed by walking?

Words that can be used to identify operations:

Addition	Subtraction	Multiplication	Division	Equals
sum	difference	product	quotient	is
total	minus	times	divide	is the same as
increased by	less than	double	per	equals
plus	more than	triple	divided equally	equal to
added to	decrease	of	divided by	results in
more	loss	twice		
gain	fewer			

Strategies for Solving Fraction Problems

The following are some strategies that may be useful when solving fraction problems that involve finding a fractional part of something and when multiplication and/or division is necessary. It should be stressed that there is no recipe for problem solving. All of us, teachers included, have had difficulty at some time when dealing with word problems. However, there are at least two things that successful problem solvers have in common: they read every problem at least twice, and they organize their work in a logical manner.

A. Adding and Subtracting Problems

1. Numbers are added when we need to combine values. The following words suggest combining values:

Combined	Altogether
In total	Sum

2. Numbers are subtracted when we need to compare values. The following words suggest comparing values:

More (than)	Less (than)
Difference	Before
After	Faster, slower, longer
Shorter, higher, lower	Remained
Left	

Examples:

1. The Carter family (Jack, June, Jill, and Jon) all share one car. During the last month, each of them put ¾ of a tank of gasoline into the car. How many full tanks of gas were put into the car?

   Solution: $\frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} = \frac{12}{4} = 3$

   Answer: The Carter family put 3 full tanks of gas into the car.

2. Last week, Bill ate pizza every day. He ate the following fractional parts of a pizza: ¾, ½, ½, ¾, ¾, ½, and ¾. How many pizzas did he eat during the week?

   Solution: $\frac{3}{4} + \frac{1}{2} + \frac{1}{2} + \frac{3}{4} + \frac{3}{4} + \frac{1}{2} + \frac{3}{4} = \frac{3}{4} + \frac{2}{4} + \frac{2}{4} + \frac{3}{4} + \frac{3}{4} + \frac{2}{4} + \frac{3}{4}$

   $= \frac{18}{4} = 4\frac{2}{4} = 4\frac{1}{2}$

   Answer: Bill ate 4½ pizzas during the week.

3. Ms. Sun had 3¾ chocolate bars. She gave away 1⅔ bars. How many chocolate bars does she have left?

   Solution: $3\frac{3}{4} - 1\frac{2}{3} = 3\frac{9}{12} - 1\frac{8}{12} = 2\frac{1}{12}$

   Answer: Ms. Sun has 2 $^{1}/_{12}$ chocolate bars left.

Exercise 4.2

The solutions to the problems below can be found by adding, subtracting, or both.

1. Three friends bought two large pizzas. One person ate ½ a pizza, another ate ⅔ of a pizza, and the final person ate $^{5}/_{6}$ of a pizza. How much pizza did they eat altogether?

2. A movie ticket costs \$8½, popcorn costs \$2¼, and a drink costs \$1¾. How much does it cost to go to a movie and have popcorn and a drink?

3. A recipe calls for 3½ cups of flour. If you only have 1¾ cups of flour, how much flour do you have to borrow from your neighbour?

4. Sandy is putting trim around the edge of a triangular pillow. Two sides of the pillow measure 20¾ cm each, and the third side measures 12⅜ cm. How much trim does Sandy need altogether?

5. The High Level Bridge in Edmonton is $1\frac{1}{5}$ km long. A city block is about $\frac{1}{10}$ of a km long. How far would a person have travelled if he walked two blocks to the bridge and then across the bridge?

6. Twins were born. The larger twin weighed 6¾ pounds, and the smaller twin weighed 5¾ pounds. How much did they weigh in total?

7. A student spent $3\frac{3}{4}$ hours in class and $2\frac{2}{3}$ hours studying. How much more time did the student spend in class than studying?

8. On Monday, a doctor worked $3\frac{1}{2}$ hours in the morning and 5 hours in the afternoon. On Tuesday, the doctor worked $3\frac{3}{4}$ hours in the morning and $4\frac{1}{2}$ hours in the afternoon. How many more hours did the doctor work on Monday than on Tuesday?

9. Last year Robert grew $7\frac{5}{8}$ inches. This year he grew $2\frac{5}{6}$ inches. How much more did he grow last year than this year?

10. A family bought 3 equal-sized pizzas. One pizza was cut into 8 pieces, and 7 pieces were eaten. A second pizza was cut into 6 pieces and only 1 piece was eaten. The third pizza was cut into 6 pieces and 4 pieces were eaten. How much pizza was eaten altogether?

11. In math, all of something is represented by the number 1. If $\frac{3}{4}$ of a football game has been played, what fraction of the game remains to be played?

12. Each month, a truck driver spends $\frac{3}{5}$ of his money on groceries and bills, $\frac{3}{20}$ of his money on entertainment, and saves the rest. What fraction of all his money does he save?

13. A nurse worked 2¾ shifts one week, 3½ shifts a second week, and 5⅜ shifts a third week. How many shifts did she work altogether?

14. If the nurse mentioned in the previous question was supposed to work 4 shifts per week, how many shifts altogether was she short of her requirement?

B. Problems Involving Multiplication

When the word ***of*** appears in a problem that requires you to find a fractional part of something, it signals multiplication.

Examples:

1. A bingo player won $100. She kept ¾ **of** her winnings and spent the rest on more bingo cards. How much money did she keep?

Solution: The keyword *of* appears in the second sentence. Start the solution by writing a brief statement summarizing the information in the *of* sentence:

She kept ¾ **of** <u>her winnings</u>

You know *of* can be replaced with a × sign   You know her winnings are $100

Now you can write a number sentence and calculate the answer.

$$\frac{3}{4} \times \$100 = \frac{3}{\cancel{4}_1} \times \frac{\cancel{100}^{25}}{1} = \frac{75}{1} = 75$$

Answer: She kept $75.

2. Bill wanted to strengthen the coolant in his Toyota by adding 2½ litres (L) of antifreeze to the radiator. In the process, he spilled ⅔ of it. How many litres of antifreeze was lost?

Solution: Here there are three sentences containing the word *of*. Which one is the key that will help us decide what to do? Study each sentence carefully to find its full meaning.

- **Sentence 3** asks the question and tells us what to find—the amount of antifreeze lost.
- **Sentence 1** tells us the amount of antifreeze Bill had to start with—2½ L.
- **Sentence 2** tells us Bill spilled (or lost) ⅔ of the antifreeze he had at the start.

Sentence 2 is the **key sentence** since it gives us information about the amount of antifreeze lost, which is what we are trying to find.

The summary statement from Sentence 2 is "He lost ⅔ of it" → You know that *it* refers to the 2½ L of antifreeze, the amount Bill had to start with.

$$\frac{2}{3} \times 2\frac{1}{2} = \frac{\cancel{2}^1}{3} \times \frac{5}{\cancel{2}_1} = \frac{5}{3} = 1\frac{2}{3}$$

Answer: Bill lost 1⅔ L of antifreeze.

3. A cyclist training for the Summer Games rode his bicycle $^5/_6$ kilometre (km) in 1 minute. A second cyclist rode only ⅔ as far as the first one. How far did the second cyclist ride?

Solution: Here we are asked to find the **distance** ridden by the second cyclist. We know this distance is ⅔ **as far as** the distance ridden by the first cyclist. Rewording this we could say:

The second cyclist rode ⅔ **of** the distance ridden by the first.

The number sentence is now clear:

Distance ridden by the second cyclist: $\frac{2}{3} \times \frac{5}{6}$

$$= \frac{\cancel{2}^1}{3} \times \frac{5}{\cancel{6}_3} = \frac{5}{9}$$

Answer: The cyclist rode $^5/_9$ km.

C. Problems Involving Division

- ✦ Problems that ask "How big is **each** part?" "How much will **each** receive?" or "How big is **each** person's share?" all signal division.
- ✦ Problems that ask how many times a smaller thing is **contained** in a larger thing also signal division.
- ✦ Problems that use the word **per** often (though not always) require division.

Examples:

1. In total, a family of 6 children collected 3½ kilograms (kg) of Halloween candy. Their mother asked them to share the candy equally. How many kilograms of candy did each child get?

Solution: It is the candy that is being shared (divided) equally among the children. Since we have 3½ kg of candy, it is this amount that must be divided equally.

Each child gets 3½ kg ÷ 6 = ? kg

$$3\frac{1}{2} \div 6 = \frac{7}{2} \div \frac{6}{1} = \frac{7}{2} \times \frac{1}{6} = \frac{7}{12}$$

Answer: Each child gets $\frac{7}{12}$ kg of candy.

2. Superstore buys walnuts by the barrel, then puts them in 1¼ kg bags for sale to the public. How many bags would be needed to prepare a shipment of walnuts that weighed 280 kg for sale?

Solution: In this problem, the question is really how many times is 1¼ (the smaller thing) contained in 280 (the larger thing)? We must divide the larger thing by the smaller thing.

Number of bags needed = 280 kg ÷ 1¼ kg

$$\frac{280}{1} \div \frac{5}{4} = \frac{\cancel{280}^{56}}{1} \times \frac{4}{\cancel{5}_{1}} = 224$$

Answer: 224 bags are needed for the shipment.

3. A shopper purchased a 5½ kg turkey for $22. How much did the turkey cost per kg?

Solution: We are asked to find the **cost** (in dollars) per (for each) **kg**. First write what is before the word *per* on the left of the division sign.

Cost ÷ (In the sentence with the question, the word **cost** appears before "per")

Next write what is **after** the word *per* on the right of the division sign.

cost ÷ kg (**kg** appears after *per*)

Now write a number sentence using information from the problem that tells about cost and kg.

The cost per kg is $22 ÷ 5½ kg

$$\frac{22}{1} \div \frac{11}{2} = \frac{\cancel{22}^{2}}{1} \times \frac{2}{\cancel{11}_{1}} = 4$$

Answer: The turkey cost $4 per kg.

Exercise 4.3

Solve the following word problems. Show your work and write your answer in a complete sentence.

1. Carol ordered 8 pizzas for her children. The children ate $\frac{1}{4}$ of the pizzas. How many pizzas are left?

2. What number subtracted from $\frac{5}{8}$ is $\frac{1}{2}$?

3. Todd spent $\frac{3}{5}$ hours tuning up his car. He then spent 30 minutes painting a door. On what job did Todd spend more of his time? By what fraction of an hour?

4. A house worth $379 500 is sold for $\frac{2}{3}$ of its value. How much is the selling price?

5. A television that normally costs $600 was sold for $\frac{1}{3}$ off the normal price.

   a. How much was taken off?

   b. How much was the television sold for?

6. The price of a house in 1977 was $35 000. In 1978, the price was $\frac{1}{5}$ higher.

   a. How much did the price increase in one year?

   b. How much did the house cost in 1978?

7. Jack and Dani are a husband and wife who both work and whose combined yearly income is
   $50 000. Jack makes $\frac{2}{5}$ of the combined income and Dani makes $\frac{3}{5}$.

   a. How much money does Jack make?

   b. How much money does Dani make?

8. A bus travelled ⅝ of a 640-kilometre trip on Monday, and the remaining ⅜ of the trip on Tuesday.

   a. How many kilometres did the bus travel on Monday?

   b. How many kilometres did it travel on Tuesday?

9. Double this recipe for cookies (multiply each amount by 2).

1 cup sugar	⅜ cup peanut butter	_____ cup sugar	_____ cup peanut butter
¼ cup butter	1⅓ cup rolled oats	_____ cup butter	_____ cup rolled oats
⅓ cup milk	¾ cup peanuts	_____ cup milk	_____ cup peanuts
½ tsp. vanilla		_____ tsp. vanilla	

10. A family spends about $1200 per year on clothing. Medication is about ¼ of the cost of clothing. How much money does the family spend on medication per year?

11. A family spends an average of $75 per month for their total telephone charges. One-third of this is for phone rental and local calls. How much per month is spent on long-distance calls?

12. A digital camera that regularly sells for $60.00 is now on sale at ¼ off. What is the sale price?

13. A car travelling from City A to City B, which is 480 kilometres away, ran out of gas ⅓ of the way through the trip. How many kilometres did the car travel before it ran out of gas?

14. Paul wants to buy a cellphone priced at $560. He pays ⅖ of the price in cash and the rest in 6 equal monthly installments. How much must he pay each month?

15. A family budgets ¼ of its annual income of $48 400 for food, $\frac{2}{5}$ for shelter (including operating expenses and furnishing). How much is budgeted for each item annually?

16. If ¾ of a pie is divided evenly among 5 people, what fraction of the pie does each person get?

Post-Module Assessment and Glossary

Post-Module Assessment

Now that you have completed this module, reassess what you can do against this checklist:

In this module I will learn how to …	I can't do this	I can do this with help	I can do this!
1. Use fractions in everyday math; solve fraction-related problems			
2. Add and subtract all types of fractions			
3. Multiply and divide all types of fractions			

Glossary for this Module

Complex fraction	A fraction formed of two fractional expressions, one on top of the other.
Denominator	Lower (bottom) part of a fraction; tells how many equal parts a whole has been divided into
Fraction	One or more equal parts of anything
***Like* fractions**	Fractions that have the same denominator or common denominators
Numerator	Upper (top) part of a fraction; tells how many parts are being considered
Reciprocal	The number by which a given number must be multiplied to get a result of one. For example, the *reciprocal* of ½ is 2 ($\frac{1}{2}$ and $\frac{2}{1}$).
***Unlike* fractions**	Fractions that do not have the same denominator

Appendix: Exercise Answer Key

Unit 1: Introduction to Fractions

Exercise 1.1

2. One third; 1 of 3 equal parts

3. One sixth; 1 of 6 equal parts

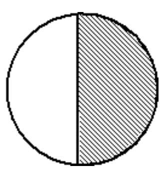
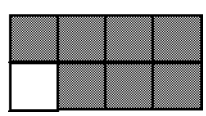
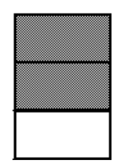
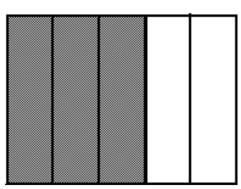
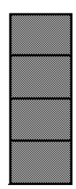
4. Three quarters or three fourths; 3 of 4 equal parts

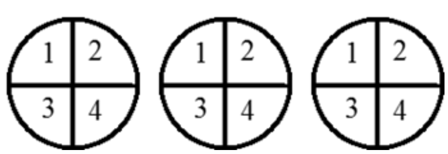
5. Two thirds; 2 of 3 equal parts

6. a. 1 of 5 equal parts b. 1 of 8 equal parts
   c. 1 of 6 equal parts d. 6 of 8 equal parts
   e. 3 of 4 equal parts f. 3 of 8 equal parts

7. a. $\frac{3}{4}$ b. $\frac{3}{8}$ c. $\frac{1}{2}$ d. $\frac{2}{3}$
   e. $\frac{2}{4}$ or $\frac{1}{2}$ f. $\frac{1}{6}$ g. $\frac{2}{3}$ h. $\frac{5}{9}$

8. a. $\frac{3}{5}$ b. $\frac{5}{6}$ c. $\frac{4}{4}$ or 1 d. $\frac{1}{5}$
   e. $\frac{3}{4}$ f. $\frac{2}{2}$ or 1 g. $\frac{2}{3}$

9. a. 
   b. 
   c. 
   d. 
   e. 

10. 3/4 

11. a. $\frac{1}{2}$ b. $\frac{1}{4}$ c. $\frac{1}{8}$ d. $\frac{1}{16}$
    e. The denominator gets larger because the whole is divided into more parts.

12. $\frac{8}{12}$ or $\frac{2}{3}$

Exercise 1.2

1. Proper 2. Improper 3. Mixed 4. Improper 5. Improper 6. Improper

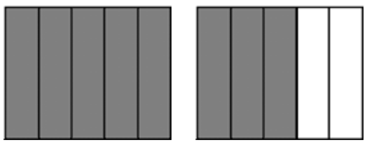
7. Mixed 8. Improper 9. Improper 10. Mixed 11. Improper 12. Proper

Exercise 1.3

1. a. $3\frac{1}{4}$ and $\frac{13}{4}$ b. $2\frac{1}{2}$ and $\frac{5}{2}$ c. $1\frac{2}{5}$ and $\frac{7}{5}$ d. $1\frac{2}{5}$ and $\frac{7}{5}$

   e. $2\frac{4}{9}$ and $\frac{22}{9}$ f. $1\frac{2}{3}$ and $\frac{5}{3}$ g. $2\frac{7}{8}$ and $\frac{23}{8}$

2. a.



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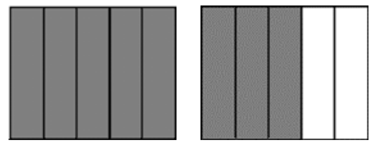

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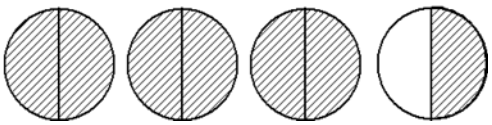

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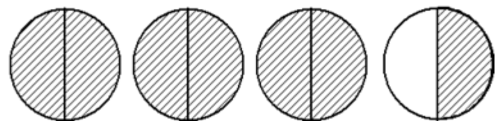
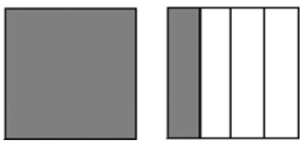


e.



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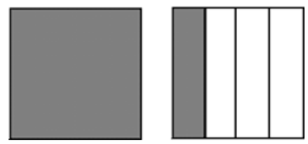


Exercise 1.4

1. $1\frac{1}{2}$ 2. $1\frac{1}{3}$ 3. $5\frac{2}{5}$ 4. 5

5. $4\frac{5}{7}$ 6. $5\frac{2}{3}$ 7. $5\frac{1}{4}$ 8. 7

Exercise 1.5

1. a. $\frac{14}{5}$ b. $\frac{33}{4}$ c. $\frac{67}{8}$ d. $\frac{21}{3}$ e. $\frac{23}{3}$ f. $\frac{16}{2}$

2. a. Mixed; $\frac{15}{4}$ b. Improper; $4\frac{1}{3}$ c. Mixed; $\frac{59}{7}$

   d. Mixed; $\frac{23}{3}$ e. Improper; $1\frac{3}{5}$ f. Mixed; $\frac{39}{4}$

3. Andrew packed $\frac{253}{10}$ or $25\frac{3}{10}$ bags.

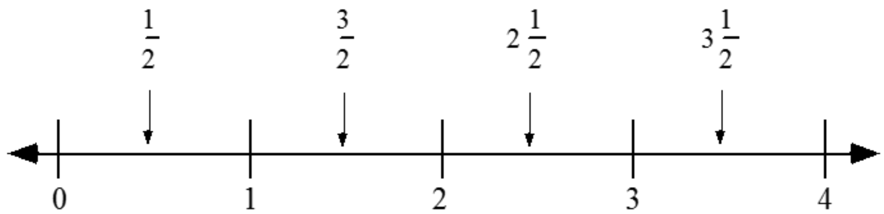
4. a. The Earth turns $\frac{25}{24}$ or $1\frac{1}{24}$ times in 25 hours.

   b. The Earth turns $\frac{55}{24}$ or $2\frac{7}{24}$ times in 55 hours.

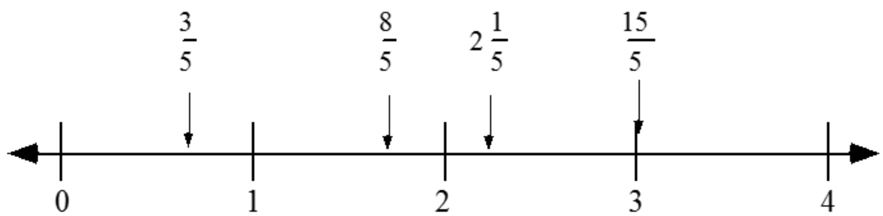
5. 197 minutes written as a mixed number is $3\frac{17}{60}$.

Exercise 1.6

1. a.



b.



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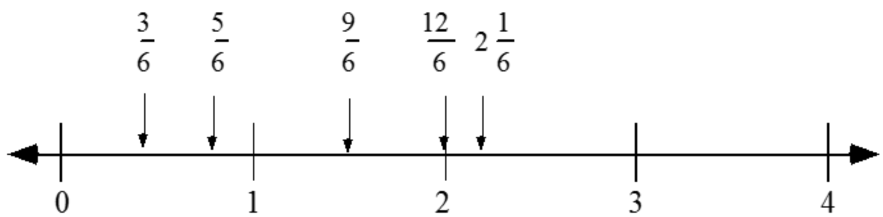
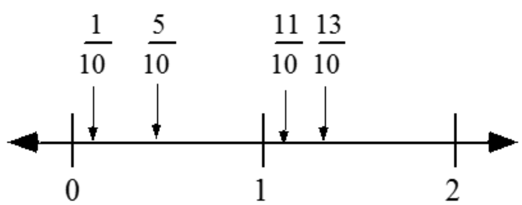


d.



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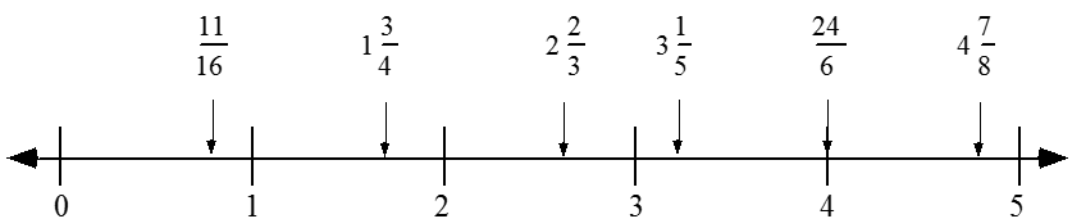
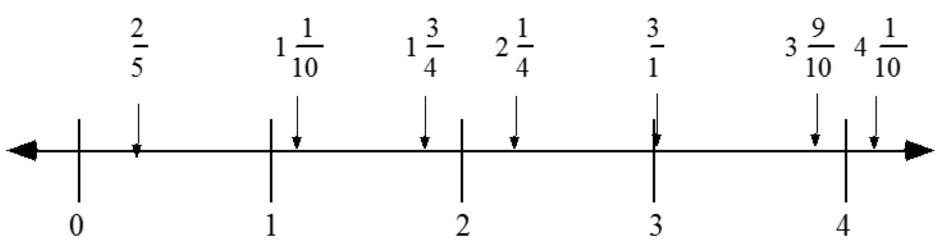


3.



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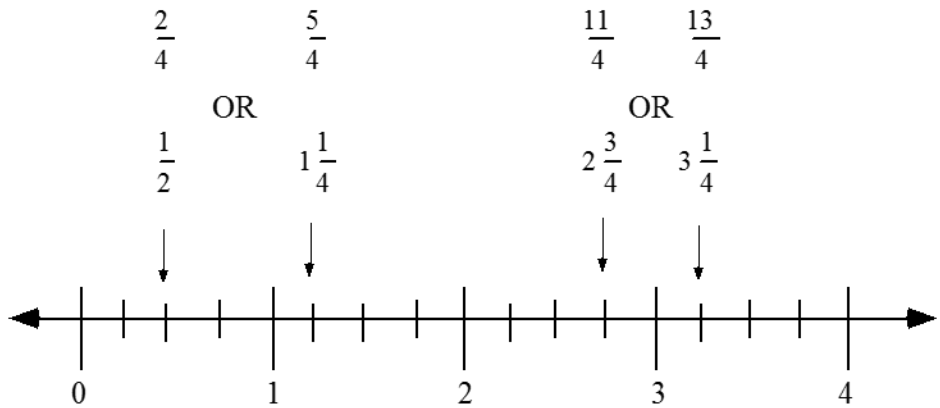
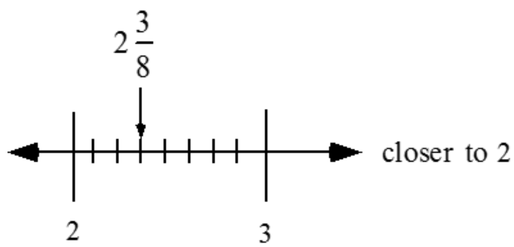
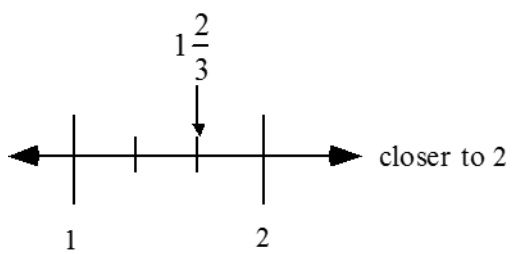


Exercise 1.7

1. a.



b.



2. a. 4 b. 4 c. 9 d. 6 e. 7

f. 9 g. 14 h. 10 i. 8 j. 12

Exercise 1.8

1. 7 2. 28 3. 14 4. 15 5. 12 6. 18

Exercise 1.9

1. 1, 2, 4; GCF 4
2. 1, 2, 4, 8; GCF 8
3. 1, 2, 3, 6; GCF 6
4. 1, 3, 9; GCF 9
5. 1, 5; GCF 5
6. 1, 2, 5, 10; GCF 10
7. 1, 5, 25; GCF 25
8. 1, 2, 4, 28; GCF 28

Exercise 1.10

1. $2 \times 2 \times 3$ or $2^2 \times 3$
2. $2 \times 3 \times 3$ or 2×3^2
3. $2 \times 2 \times 3 \times 3$ or $2^2 \times 3^2$
4. $2 \times 5 \times 7$
5. $3 \times 3 \times 7$ or $3^2 \times 7$
6. $3 \times 3 \times 5$ or $3^2 \times 5$
7. $3 \times 3 \times 3 \times 3$ or 3^4
8. $2 \times 3 \times 3 \times 5$ or $2 \times 3^2 \times 5$
9. $2 \times 2 \times 5 \times 5$ or $2^2 \times 5^2$
10. $2 \times 2 \times 5 \times 5 \times 7$ or $2^2 \times 5^2 \times 7$

Exercise 1.11

1. a. ⅔ b. 3⁄5 c. 8⁄9 d. ⅔ e. ⅔
   f. 5⁄7 g. 5⁄12 h. 7⁄9 i. 9⁄10
2. a. 1 b. 1 2⁄5 c. 1⅓ d. 2½ e. 3
   f. 4 g. 3 h. 6 i. 8
3. a. 1⁄6 b. ⅓ c. ½ d. ⅔ e. 5⁄6 f. 6⁄6 = 1
4. a. A
   b. S – Only if they can be divided by a common factor
   c. S – Only if they are the same number
   d. N – A mixed number contains a whole number
   e. A – The denominator has to be larger than the numerator

Unit 2: Addition and Subtraction of Fractions

Exercise 2.1

1. a. 4⁄5 b. 1½ c. 1¼
2. a. 4⁄5 b. 1 1⁄7 c. 1 9⁄25 d. 1⅓ e. 1 3⁄16
   f. ½ g. ½ h. ⅔ i. ⅞ j. ¾
   k. 1⅔ l. 1¼

Exercise 2.2

1. $4\frac{2}{3}$	2. $6\frac{1}{5}$	3. $9\frac{3}{5}$	4. $17\frac{1}{7}$	5. $16\frac{5}{9}$
6. $19\frac{1}{4}$	7. $10\frac{1}{2}$	8. $4\frac{11}{12}$	9. $5\frac{7}{9}$	10. $28\frac{2}{3}$
11. $6\frac{1}{6}$	12. $19\frac{1}{3}$	13. $27\frac{2}{9}$	14. $11\frac{4}{5}$	15. 19

16. Joe takes $1\frac{1}{2}$ hours in all.

Exercise 2.3

1. 40	2. 36	3. 40	4. 80	5. 70
6. 140	7. 12	8. 60	9. 126	10. 60

Exercise 2.4

1. a. $1\frac{1}{2}$	b. $1\frac{19}{36}$	c. $\frac{53}{60}$
2. a. $1\frac{2}{15}$	b. $1\frac{13}{30}$	c. $1\frac{1}{4}$
3. a. $1\frac{7}{40}$	b. $2\frac{5}{18}$	c. $1\frac{17}{45}$
4. a. $\frac{13}{16}$	b. $\frac{1}{3}$	c. $\frac{23}{30}$
5. a. $1\frac{7}{36}$	b. $\frac{29}{35}$	c. $1\frac{1}{36}$
6. a. $\frac{5}{6}$	b. $\frac{1}{2}$	c. $\frac{23}{48}$

7. Andrew has dug $\frac{11}{12}$ of the way and has $\frac{1}{12}$ left to go.

Exercise 2.5

1. 3; $3\frac{5}{8}$	2. 7; $7\frac{5}{16}$	3. 11; $11\frac{1}{10}$	4. 9; $9\frac{5}{8}$
5. 11; $10\frac{1}{16}$	6. 14; $14\frac{1}{12}$	7. 36; $36\frac{11}{40}$	8. 16; $15\frac{1}{4}$
9. 13, $12\frac{7}{20}$	10. 27; $26\frac{11}{12}$	11. 25; $24\frac{1}{4}$	12. 10; $11\frac{1}{16}$
13. 39; $38\frac{31}{60}$	14. 56; $55\frac{1}{12}$	15. 63; $62\frac{11}{36}$	16. 42; $42\frac{2}{5}$

Exercise 2.6

1. $\frac{1}{2}$	2. $\frac{2}{3}$	3. $\frac{0}{8} = 0$	4. $\frac{1}{4}$	5. $\frac{2}{5}$
6. $\frac{1}{2}$	7. $\frac{1}{3}$	8. $\frac{1}{2}$	9. $\frac{2}{5}$	

Exercise 2.7

1. a. $\frac{1}{6}$	b. $\frac{1}{4}$	c. $\frac{1}{6}$	d. $\frac{1}{10}$	e. $\frac{1}{8}$	f. $\frac{5}{16}$
2. a. $\frac{3}{16}$	b. $\frac{1}{5}$	c. $\frac{3}{10}$	d. $\frac{11}{40}$	e. $\frac{1}{4}$	f. $\frac{3}{4}$
g. $\frac{7}{30}$	h. $\frac{7}{16}$	i. $\frac{1}{24}$	j. $\frac{1}{16}$	k. $\frac{11}{24}$	l. $\frac{1}{6}$

Exercise 2.8

1. $1\frac{1}{4}$	2. $2\frac{1}{3}$	3. $1\frac{1}{2}$	4. $1\frac{1}{2}$	5. $2\frac{2}{3}$	6. $1\frac{1}{2}$
7. $\frac{1}{8}$	8. $5\frac{4}{5}$	9. $2\frac{3}{4}$	10. $4\frac{1}{6}$	11. $1\frac{3}{5}$	12. $\frac{2}{3}$

Exercise 2.9

1. $1\frac{1}{2}$	2. $3\frac{2}{3}$	3. $6\frac{1}{5}$	4. $5\frac{3}{4}$	5. $2\frac{1}{2}$	6. $3\frac{7}{8}$
7. $10\frac{7}{24}$	8. $5\frac{11}{12}$	9. $3\frac{5}{28}$	10. $1\frac{5}{8}$	11. $1\frac{19}{24}$	12. $11\frac{13}{20}$
13. $6\frac{13}{28}$	14. $4\frac{5}{18}$	15. $12\frac{1}{6}$	16. $5\frac{23}{40}$	17. $10\frac{17}{36}$	18. $3\frac{8}{15}$
19. $2\frac{1}{12}$	20. $6\frac{1}{12}$	21. $\frac{11}{12}$	22. $14\frac{4}{5}$	23. $11\frac{23}{40}$	24. $2\frac{31}{40}$

Unit 3: Multiplication and Division of Fractions

Exercise 3.1

1. $\frac{1}{2}$	2. $\frac{4}{27}$	3. $\frac{1}{3}$	4. $\frac{7}{12}$	5. $\frac{1}{24}$
6. $\frac{21}{32}$	7. $\frac{35}{36}$	8. $\frac{21}{10}$	9. $\frac{3}{35}$	10. $\frac{3}{10}$
11. $\frac{5}{4}$	12. $\frac{3}{10}$	13. $\frac{2}{11}$	14. $\frac{1}{2}$	15. $\frac{8}{35}$
16. $\frac{7}{10}$	17. $\frac{5}{8}$	18. $\frac{2}{9}$	19. $\frac{1}{2}$	20. $\frac{1}{3}$

Exercise 3.2

1. 75	2. 50	3. $13\frac{1}{2}$	4. $\frac{7}{10}$	5. 20
6. $4\frac{2}{3}$	7. 12	8. $2\frac{2}{5}$	9. $1\frac{11}{14}$	10. $24\frac{2}{5}$

11. a. Joe takes 4½ minutes to walk to school.
    b. He spends 45 minutes each week walking to and from school.
12. Tony has 9¼ pages left to read.
13. Debra needs 6¼ cups of flour, 4⅜ cups of sugar, and ⅝ cup of baking powder.

Exercise 3.3

1. $\frac{9}{1}$	2. $\frac{17}{6}$	3. $\frac{11}{12}$	4. $\frac{5}{22}$	5. $\frac{1}{14}$	6. $\frac{9}{10}$
7. $\frac{15}{49}$	8. $\frac{26}{57}$	9. $\frac{3}{40}$	10. $\frac{22}{5}$	11. ½	12. $\frac{4}{5}$
13. $\frac{3}{50}$	14. $\frac{61}{100}$	15. $\frac{20}{9}$	16. 1 or $\frac{1}{1}$		

Exercise 3.4

1. 1	2. 1½	3. $1\frac{2}{9}$	4. 44	5. 20
6. ⅛	7. ¾	8. ¼	9. 15	10. ⅔
11. 141	12. 2	13. $\frac{3}{7}$	14. $\frac{4}{5}$	15. $1\frac{5}{12}$

16. Cecil could share with 5 friends.
17. a. Each person will run 1⅓ times around the track.
    b. Each person will take 27⅓ seconds.
    c. The team needs 109⅓ seconds to complete the relay.

Exercise 3.5

1. $\frac{1}{12}$	2. 6	3. $\frac{1}{32}$	4. 3
5. $\frac{4}{5}$	6. $\frac{4}{9}$	7. $\frac{1}{14}$	8. 1½
9. 16	10. $\frac{1}{6}$	11. 32	12. ⅓

Exercise 3.6

1. 3	2. 20	3. $16\frac{1}{2}$	4. $\frac{1}{16}$	5. $\frac{1}{6}$
6. $\frac{3}{5}$	7. $\frac{8}{15}$	8. $\frac{1}{3}$	9. $\frac{3}{25}$	10. $4\frac{4}{5}$
11. $\frac{2}{3}$	12. $2\frac{1}{10}$	13. $\frac{3}{10}$	14. $2\frac{2}{5}$	

Unit 4: Fraction Word Problems

Exercise 4.1

1. Joel has planted $\frac{13}{32}$ of the garden in corn.
2. Lin missed $\frac{1}{5}$ of the school week. She attended 4/5 of the week.
3. $\frac{4}{9}$ of the starters have beards. $\frac{5}{9}$ of the starters don't have beards.
4. Ahmed drives ¾ of the distance. He walks ¼ of the distance.
5. $\frac{1}{6}$ of Mary's family likes to play tennis. $\frac{5}{6}$ of the family does not.
6. Lucia spilled $\frac{119}{250}$ of the drink. She has $\frac{131}{250}$ left.
7. $\frac{5}{12}$ of the eggs were broken. $\frac{7}{12}$ of the eggs were not broken.
8. The pothole brigade has $\frac{19}{24}$ of the potholes left to fill.
9. a. $\frac{2}{7}$ of the rooms are bathrooms.
   b. The kitchen represents $\frac{1}{7}$ of the rooms.
   c. $\frac{3}{7}$ of the rooms are bedrooms.
10. $\frac{341}{400}$ of the books contains only printed material.
11. a. $\frac{8}{19}$ of the class is female.
    b. $\frac{17}{19}$ of the class is between 19 and 35 years old.
    c. $\frac{11}{19}$ of the class is male.
12. a. $\frac{10}{21}$ of the journey was completed by bus.
    b. $\frac{3}{7}$ of the journey was completed by car.
    c. $\frac{2}{21}$ of the journey was completed by walking.

Exercise 4.2

1. They ate 2 pizzas altogether.
2. It costs $12½ to go to a movie and have popcorn and a drink.
3. You have to borrow 1¾ cups of flour from your neighbour.
4. Sandy needs 53⅞ cm of trim altogether.
5. He would have travelled $1\frac{2}{5}$ km.
6. The twins weighed 12½ pounds in total.
7. He spent $1\frac{1}{12}$ hours more time in class than studying.
8. The doctor worked ¼ hour more on Monday than on Tuesday.
9. He grew $4\frac{19}{24}$ inches more last year than this year.
10. $1\frac{17}{24}$ pizzas were eaten altogether.
11. ¼ of the game remains to be played.
12. The truck driver saves ¼ of his money.
13. The nurse worked 11⅝ shifts altogether.
14. She was ⅜ of a shift short of her requirement.

Exercise 4.3

1. There are 6 pizzas left.
2. ⅛ subtracted from ⅝ is ½.
3. Todd spent $\frac{1}{10}$ of an hour more tuning up his car.
4. The selling price is $253 000.
5. a. $200 was taken off.
   b. The television sold for $400.
6. a. The price of the house increased $7000.
   b. The house cost $42 000 in 1978.
7. a. Jack makes $20 000.
   b. Dani makes $30 000.

8. a. The bus travelled 400 km on Monday.
   b. The bus travelled 240 km on Tuesday.

9. 2 cups sugar, $\frac{3}{4}$ cups peanut butter, $\frac{1}{2}$ cup butter, $2\frac{2}{3}$ cups rolled oats, $\frac{2}{3}$ cups milk, $1\frac{1}{2}$ cups peanuts, and 1 tsp vanilla

10. They spend $300 on medication.

11. They spend $50 a month on long-distance calls.

12. The sale price is $45.

13. The car travelled 160 km before it ran out of gas.

14. He must pay $56 per month.

15. The budget is $12 100 for food and $19 360 for shelter.

16. Each person gets $\frac{3}{20}$ of the pie.