

Foundational Numeracy

-12°C

Module 6: Integers, including Fractions and Decimals

Solutions Manual

Developed for Alberta's Community Adult Learning Program



Funded by Alberta Advanced Education



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Contents

Introduction to the Module	1
Specific Learning Outcomes.....	1
Essential Skills.....	2
Unit 1: Introduction to Signed Numbers	3
Introductory Video Link to Understanding Integers	3
Think about	3
Keywords.....	3
Integers on a Number Line	3
Absolute Value	4
Exercise 1.1	5
Exercise 1.2	6
Unit 2: Addition and Subtraction of Integers	7
Keywords.....	7
Adding Integers	7
Summary of Addition Rules.....	9
Exercise 2.1	10
Subtracting Integers.....	12
Summary of Subtraction Rules.....	13
Exercise 2.2	15
Problem Solving with Integers	16
Tips for Solving Application Problems.....	16
Exercise 2.3	17
Unit 3: Multiplication and Division of Signed Numbers	19
Think about.....	19
Keywords.....	19
Multiplying Integers	19
Summary of Multiplication Rules	20
Exercise 3.1	21
Powers and Exponents, and Square Roots of Signed Numbers.....	22
Keywords.....	22

Powers	22
Square Root of a Number	23
Exercise 3.2	24
Division of Signed Numbers	25
Summary of Division Rules	25
Exercise 3.3	26
Order of Operations with Integers	27
Think about	27
Keywords.....	27
Order of Operations with Whole Numbers	27
Order of Operations Rules.....	28
Exercise 3.4	31
Order of Operations with Integers	33
Exercise 3.5	34
Problem Solving	35
Tips for Solving Application Problems.....	35
Exercise 3.6	36
Unit 4: Operations with Signed Decimals.....	37
Adding and Subtracting Decimals	37
Adding Signed Decimals.....	37
Subtracting Signed Decimals	37
Exercise 4.1	38
Multiplying and Dividing Signed Decimals	39
Multiplication Rules of Signs.....	39
Division Rules of Signs.....	39
Exercise 4.2	40
Order of Operations with Decimals.....	42
Exercise 4.3	42
Unit 5: Operations with Signed Fractions.....	43
Adding and Subtracting Signed Fractions	43
Adding Signed Fractions.....	43
Subtracting Signed Fractions.....	44
Exercise 5.1	45

Multiplying and Dividing Signed Fractions	47
Multiplying Signed Fractions	47
Dividing Signed Fractions	48
Exercise 5.2	49
Order of Operations with Fractions	51
Exercise 5.3	52
Post-Module Assessment and Glossary	55
Post-Module Assessment	55
Glossary for this Module	55
Appendix: Exercise Answer Key	57
Unit 1: Introduction to Signed Numbers	57
Exercise 1.1	57
Exercise 1.2	57
Unit 2: Addition and Subtraction of Integers	57
Exercise 2.1	57
Exercise 2.2	57
Exercise 2.3	58
Unit 3: Multiplication and Division of Signed Numbers.....	58
Exercise 3.1	58
Exercise 3.2	58
Exercise 3.3	58
Exercise 3.4	58
Exercise 3.5	59
Exercise 3.6	59
Unit 4: Operations with Signed Decimals	59
Exercise 4.1	59
Exercise 4.2	59
Exercise 4.3	59
Unit 5: Operations with Signed Fractions	60
Exercise 5.1	60
Exercise 5.2	60
Exercise 5.3	60

Introduction to the Module

In this module, you will solve numerical expressions using the order of operations. You will also explore signed numbers and practise solving numerical expressions with negative and positive numbers. Enjoy your studies!

Important

When you see an object like the one below, you can use a QR code scanner on your phone or tablet, and it will play a video of the math example.



Specific Learning Outcomes

The table below displays the skills and knowledge you will explore in this module. This is your opportunity to evaluate your own skills to see if you can do these things. At the end of this module, you will be invited to re-evaluate your skills to measure the progress you have made.

In this module, I will learn how to ...	I can't do this	I can do this with help	I can do this!
1. Solve numerical expressions with negative and positive numbers			
2. Solve numerical expressions using the order of operations			

Essential Skills

The following essential skills are used in this module:



Reading: Understanding materials written in sentences or paragraphs



Numeracy: Using and understanding numbers



Computer use/digital technology: Using computers and other tools such as calculators and phones



Vocabulary: Gaining related vocabulary



Thinking: Finding information, problem solving, decision making, using your memory, planning job tasks and being organized, critical thinking

Unit 1: Introduction to Signed Numbers

Introductory Video Link to Understanding Integers



Think about ...

- ◆ What is a negative number?
- ◆ What is the difference between -20°C and 20°C ?

Keywords

Integer	A number that is written without a fractional component; it can be positive or negative
Negative whole numbers	Numbers less than zero, such as $-1, -2, -3, \dots$
Positive whole numbers	Numbers greater than zero, such as $1, 2, 3, \dots$

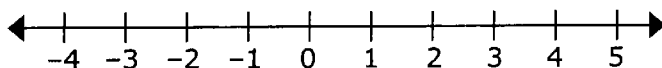
Integers on a Number Line

Numbers such as $1, 2, 3, \dots$, sometimes written $+1, +2, +3, \dots$, are called **positive whole numbers**.

Numbers such as $-1, -2, -3, \dots$ are called **negative whole numbers**. The two sets together make the set of **integers**.

There are many opposites in everyday life that use integers to describe them. Temperatures can be above freezing (positive) or below freezing (negative). Directions to the left are negative, whereas directions to the right are positive. In land elevations, positive integers indicate elevations above sea level, while negative integers indicate elevations below sea level.

Integers can be represented on a number line.



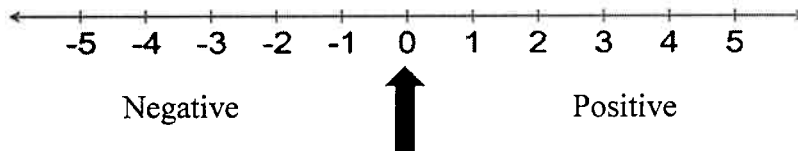
Integers to the right are larger in value. Integers to the left are smaller in value. For example, -2 is greater than -4 .

Example:

Arrange the integers -4 , 6 , 0 , -7 in order from smallest to largest.

Answer: -7 , -4 , 0 , 6

A number line is like a thermometer turned sideways.



Zero is neither negative nor positive.

Student practice:

Put the following numbers in order from smallest to largest.

-24 , -30 , -7 , -10 , -40

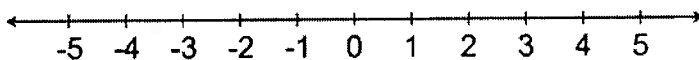
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Absolute Value

The absolute value of a number is its distance from zero. The absolute value of a number is always positive. Absolute value brackets look like $| \quad |$

The absolute value of $|2|$ is 2 and $|-2|$ is 2. Place them both on the number line.



Notice that 2 and -2 are both 2 units from zero. These numbers are called opposites as they are the same distance from but on opposite sides of zero.

Student practice:

Find the absolute value of each of the following:

1. $|-3|$

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2. $|7|$

3. $|8 - 12|$

**Exercise 1.1**

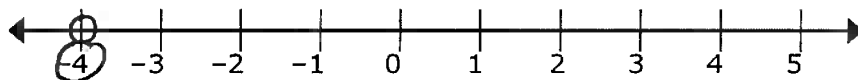
Write the following using positive and negative numbers. If the number is positive, show it in two ways.

- | | |
|----------------------------|--|
| 1. A loss of \$500 | <u>$-\\500</u> |
| 2. A gain of 300 feet | <u>$+300$ or 300</u> |
| 3. A decrease of 7 pounds | <u>-7</u> |
| 4. An increase of 3 inches | <u>$+3, 3$</u> |
| 5. A decrease of 20% | <u>-20%</u> |
| 6. 7 cars fewer | <u>-7</u> |
| 7. \$50 more | <u>$+\\$50, \\50</u> |
| 8. Down 30 metres | <u>-30</u> |
| 9. A gain of 7000 feet | <u>$+7000, 7000$</u> |
| 10. A loss of 12 yards | <u>-12</u> |

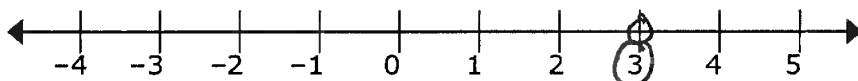
Exercise 1.2

1. Graph each number on the number line by circling the number.

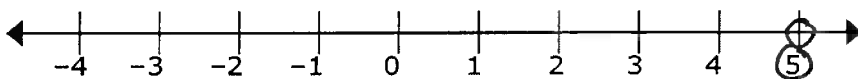
a. -4



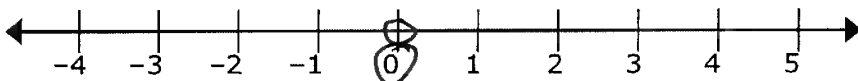
b. 3



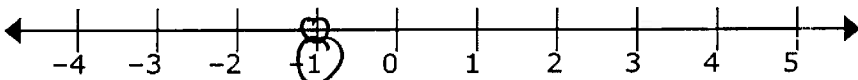
c. 5



d. 0



e. -1



2. Write $<$ or $>$ between each pair of numbers to make a true statement.

Remember that $>$ means **greater than** and $<$ means **less than**

a. $0 < 2$

b. $1 > -4$

c. $-4 < -2$

d. $-7 < -2$

e. $-8 < 2$

f. $-4 > -7$

g. $-5 < 5$

h. $4 > -12$

i. $-14 < -7$

j. $-8 > -24$

3. Write the absolute value of each of the following.

a. $|-8|$ 8

b. $|0|$ 0

c. $|15|$ 15

d. $|-37|$ 37

e. $|-218|$ 218

f. $|45|$ 45

Unit 2: Addition and Subtraction of Integers

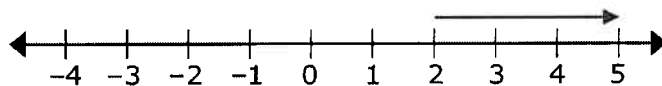
Keywords

Like signs	Signs that are the same
Unlike signs	Signs that are opposite
Subtracting integers	Add the opposite of the second number

Adding Integers

Study the following examples:

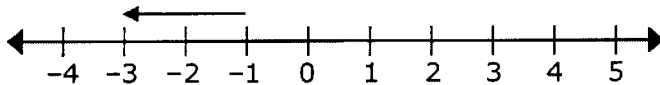
1. Add using a number line: $(+2) + (+3)$



Start at +2. Move 3 units to the **right** because 3 is positive (+3).

Answer: +5

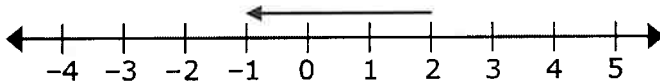
2. Add using a number line: $(-1) + (-2)$



Start at -1. Move 2 units to the **left** because 2 is negative (-2).

Answer: -3

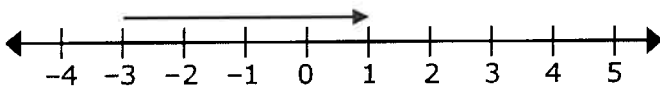
3. $(+2) + (-3)$



Start at +2. Move 3 units to the **left** because 3 is negative (-3).

Answer: -1

4. $(-3) + (+4)$



Start at -3. Move 4 units to the **right** because 4 is positive (+4).

Answer: +1

5. $(+4) + (-5) + (+6) + (-3)$

Add the positives first, then the negatives.

$$= (+4) + (+6) + (-5) + (-3)$$

$$= (+10) + (-8)$$

$$= +2$$

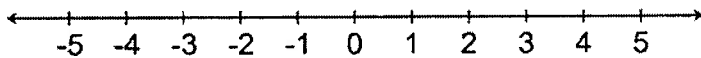
Student practice:

Add and subtract using a number line.

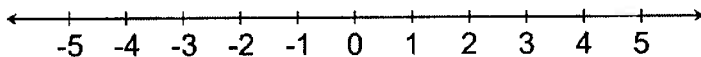
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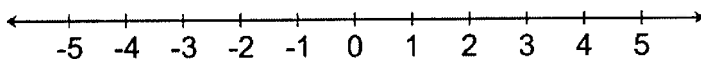
1. $2 - 3 =$



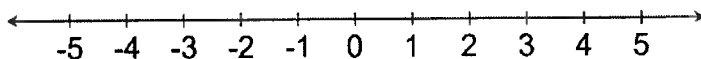
2. $-2 - 3 =$



3. $-2 + 3 =$



4. $-2 - (-3) =$



Summary of Addition Rules

When signs are the same ...

Add the absolute values of the numbers and the signs stay the same.

- **Positive + Positive = Positive** $5 + 4 = 9$
- **Negative + Negative = Negative** $(-7) + (-2) = -9$

When signs are opposite or *unlike* ...

Subtract the absolute values. The larger number keeps the sign.

- **Negative + Positive:** Use absolute values and subtract the smaller number from the larger one. The result takes the sign of the larger number.
 - $(-7) + 4 = |-7| + |4| = 7 - 4 = 3$, so -3 is your answer
 - $6 + (-9) = -3$
 - $(-3) + 7 = 4$
 - $5 + (-3) = 2$

Student practice:

1. $-15 + (-46) + (-29) =$

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2. $15 + (-46) + 29 =$



Exercise 2.1

1. $(+6) + (+2) =$

$$6 + 2 = 8$$

2. $(+7) + (+10) =$

$$7 + 10 = 17$$

3. $(-8) + (-6) = -14$

$$8 + 6 = 14$$

4. $(-2) + (-3) = -5$

$$2 + 3 = 5$$

5. $-9 + -5 =$

$$9 + 5 = 14$$

6. $-12 + -7 =$

$$12 + 7 = 19$$

7. $(+70) + (+30) =$

$$70 + 30 = 100$$

8. $-16 + (+26) =$

$$26 - 16 = 10$$

9. $134 + (-256) =$

$$256 - 134 = 122$$

10. $-169 + -234 =$

$$169 + 234 = 403$$

11. $(+16) + (-26) =$

$$26 - 16 = 10$$

12. $-12 + 25 =$

$$25 - 12 = 13$$

13. $(-18) + (+7) =$

$$18 - 7 = 11$$

14. $(34) + (-29) =$

$$34 - 29 = 5$$

15. $(+19) + (-47) + (-13) =$

$$19 + [-47 + -13]$$

$$19 + -62$$

$$62 - 19 = 43$$

16. $(-16) + (+19) + (-10) =$

$$-16 + -10$$

$$-26 + 19$$

$$26 - 19 = 7$$

$$17. \cancel{(-12)} + (+15) + (-34) + \cancel{(+12)} =$$

-12 and +12 are opposites
so can cancel

$$+15 + -34$$

$$34 - 15 = 19$$

$$18. (+24) + (-5) + (-30) + (+17) =$$

$$+24 + +17 + -5 + +30$$

$$+41 + -35$$

$$41 - 35 = 6$$

$$19. -22 + (+17) + \cancel{(-34)} + (+12) + (+8) + \cancel{+34} =$$

-34 + 34 opposite cancel

$$-22 + +17 + +12 + +8$$

$$-22 + +37$$

$$37 - 22 = 15$$

$$20. 12 + (-15) + (-27) + (+3) + (-11) + (+13) + (-14) =$$

$$+12 + +3 + +13 + -27 + -11 + -15 + -14$$

$$+28 + -77$$

$$77 - 28 = -49$$

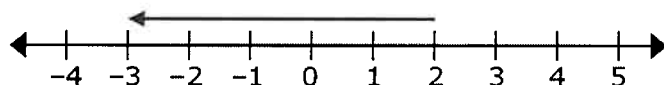
Subtracting Integers

Subtraction means changing the subtraction sign ($-$) to the addition sign ($+$), and changing the sign of the number behind it. This is known as “adding the opposite.”

Study the following examples:

1. $(+2) - (+5)$

Start at $+2$. Change the direction to -5 (with integers, subtraction means to add the opposite). Move 5 units to the **left** to **add**.

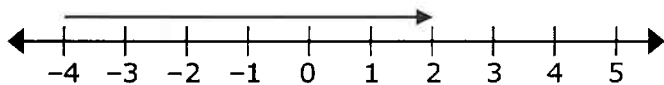


Answer: -3

Or: $(+2) - (+5) = (+2) + (-5) = -3$ ← Add the opposite

2. $(-4) - (-6)$

Start at -4 . Change the direction to $+6$. Move 6 units to the **right** to **add**.



Answer: 2 or $+2$

Or: $(-4) - (-6) = (-4) + (+6) = +2$

3. $(+6) + (-5) - (+3) - (-7)$

$= (+6) + (-5) + (-3) + (+7)$ ← Change the subtraction to addition and change the signs to their opposite.

$= (+6) + (+7) + (-5) + (-3)$ ← Add the positives, then add the negatives.

$= (+13) + (-8)$

$= +5$

Note: Addition signs followed by brackets can be dropped.

$(+6) + (-5) + (-3) + (+7)$ can be written as $6 - 5 - 3 + 7$

4. Rewrite $(-4) + (+10) - (+5) - (+2)$ without addition signs and brackets, then simplify.

$= (-4) + (+10) + (-5) + (+2)$

$= -4 + 10 - 5 + 2$

$= -9 + 12$

$= 3$

Summary of Subtraction Rules

When subtracting integers, add the first number to the **opposite** of the second number.

Change the subtraction sign to an addition sign, and change the sign of the following number to its opposite.

Remember, if the number doesn't have a sign, the number is positive.

Never subtract—always add the opposite

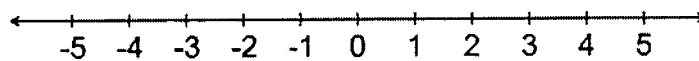
- **Negative – Positive = Negative** $(-5) - 3 = -5 + (-3) = -8$
- **Positive – Negative = Positive + Positive = Positive** $5 - (-3) = 5 + 3 = 8$
- **Negative – Negative = Negative + Positive**
Use the sign of the larger number and subtract. (Change double negatives to a positive.)
 $(-5) - (-3) = (-5) + 3 = -2$
 $(-3) - (-5) = (-3) + 5 = 2$

Student practice:

Solve the following numerical expressions by both using a number line and without using a number line.

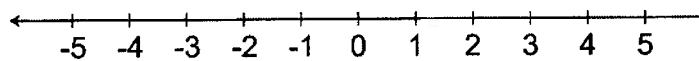


1. $2 - 3 =$

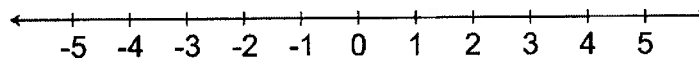


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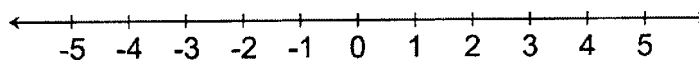
2. $-2 - 3 =$



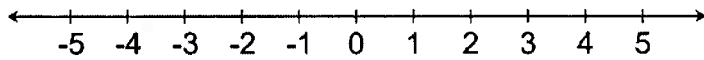
3. $-2 + 3 =$



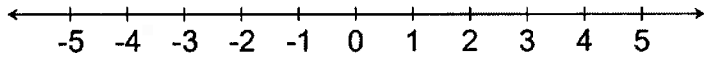
4. $-2 - (-3) =$



5. $2 - 5 =$



6. $-3 - (-7) =$



Exercise 2.2

Solve the following numerical expressions.

$$1. \quad (-1) - (+2) = -1 + -2 \\ = -3$$

$$2. \quad (+3) - (+5) = +3 + -5 \\ 5 - 3 = -2$$

$$3. \quad (-7) - (+10) = \\ = -7 + -10 = -17$$

$$4. \quad (-12) - (-8) = -12 + +8 \\ 12 - 8 = -4$$

$$5. \quad (-5) - (-2) = -5 + +2 \\ 5 - 2 = -3$$

$$6. \quad (+7) - (+10) = +7 + -10 \\ 10 - 7 = -3$$

$$7. \quad (-6) - (+8) = -6 + -8 \\ 6 + 8 = 14$$

$$8. \quad (+16) - (-16) = +16 + +16 \\ = 16 + 16 \\ = 32$$

$$9. \quad (-16) - (-16) = -16 + +16 \\ = 0$$

$$10. \quad 23 - (-35) = +23 + +35 \\ = 58$$

$$11. \quad -72 - (-29) = -72 + +29 \\ = 72 - 29 \\ = -43$$

$$12. \quad -5 - (+37) = -5 + -37 \\ = -42$$

$$13. \quad (-27) - (+25) - (-15) = \\ = -27 + -25 + +15 \\ = -52 + +15 \\ = 52 - 15 = -37$$

$$14. \quad (+11) - (-25) - (+13) = \\ +11 + +25 + -13 \\ +36 + -13 \\ 36 - 13 = 23$$

$$15. \quad (-12) - (+15) + (-34) - (-12) = \\ = -12 + -15 + -34 + +12 \\ = -15 + -34 \\ = -49$$

$$16. \quad (-16) - (+5) - (-30) - (-12) = \\ = -16 + -5 + +30 + +12 \\ = -21 + +42 \\ = 42 - 21 \\ = +21$$

Problem Solving with Integers

Tips for Solving Application Problems

Read the tips below on solving application problems.

- **Step 1:** Read the question carefully. Read the problem several times.
- **Step 2:** Work out a plan and write an equation to solve the problem.
- **Step 3:** Estimate. Is your answer reasonable?
- **Step 4:** Solve the problem. Is your answer reasonable?
- **Step 5:** Write a statement.

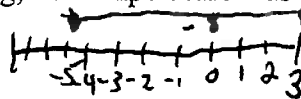
Words that can be used to identify operations:

Addition	Subtraction	Equals
sum	difference	is
total	minus	is the same as
increased by	less than	equals
plus	more than	equal to
added to	decrease	results in
more	loss	
gain	fewer	

Exercise 2.3

1. The temperature was 3°C at midday. By evening, the temperature was -5°C . What was the change in temperature?

$$-5 - 3 = -8 \text{ change}$$



2. A football team has to gain at least 10 yards (yds.) during four plays to keep the ball. Suppose that on four plays a team lost 6 yds., gained 8 yds., lost 2 yds., and gained 7 yds. Did the team gain enough yards to keep the ball?

$$\begin{aligned} & -6 + +8 + -2 + +7 \\ = & -6 + -2 + +8 + +7 \\ = & -8 + +8 + +7 \\ = & +7 \text{ yards} \end{aligned} \quad \text{No didn't keep the ball.}$$

3. On Monday, Jim had \$125 in his bank account. He deposited \$78, and paid bills of \$33 and \$142. How much money is left in his bank account?

$$\begin{aligned} & +125 + +78 + -33 + -142 \\ = & +203 + -175 \\ = & +28 \end{aligned}$$

4. The temperature was -7°C at midnight. By the next day, the temperature had risen by 11°C . What was the temperature the next day?

$$\begin{aligned} & -7 + +11 \\ = & 11 - 7 = 4 \\ = & +4^{\circ}\text{C} \end{aligned}$$

5. At 8:00 a.m., the temperature was -18 degrees C. By 10:00 a.m., the temperature was 2 degrees C higher, and by 2:00 p.m., it had increased by another 4 degrees C. By 7:00 p.m., the temperature had dropped 13 degrees C. What was the temperature at 7:00 p.m.?

$$\begin{aligned} & -18 + +2 + +4 + -13 = \\ = & -18 + -13 + +2 + +4 \\ = & -31 + +6 \\ = & -25^{\circ}\text{C} \end{aligned}$$

Unit 3: Multiplication and Division of Signed Numbers

Think about...

- ◆ How do you multiply and divide negative and positive numbers?

Keywords

Integer	A number that is written without a fractional component; it can be positive or negative
Product	The answer when two or more numbers are multiplied together
Quotient	The answer when one number is divided by another

Multiplying Integers

The **product** of two integers with *like signs* is **positive**.

$$(+4) \times (+5) = +20 \qquad (-4) \times (-5) = +20$$

The **product** of two integers with *unlike signs* is **negative**.

$$(+4) \times (-3) = -12 \qquad (-4) \times (+3) = -12$$

Study the following examples.

- $(-4) \times (+2) \times (-3)$ ← Multiply the first two factors
 $= (-8) \times (-3)$
 $= +24$

Note: If there is an **even** number of **negative factors**, the product will always be **positive**.
(In this example, there were two negative factors, so the product was positive.)

If there is an **odd** number of **negative factors**, the product is **negative**.

- $(-3) \times (+5) \times (-2) \times (-1)$
 $= (-30)$ ← There are three negative factors (an odd number), so the product will be **negative**.

When brackets are together, it means to multiply. Therefore, example 2 can also be written as

$$(-3)(+5)(-2)(-1) = -30$$

This will become especially important when you start working with formulas. For example, the area of a square is calculated as $Area = Length \times Width$ but this is most often written as $A = LW$. (When working with formulas, the brackets are implied.)

When you see brackets together, multiply!

Summary of Multiplication Rules

- **Positive \times Positive = Positive** $3 \times 2 = 6$
- **Negative \times Negative = Positive** $(-2) \times (-8) = 16$
- **Negative \times Positive = Negative** $(-3) \times 4 = -12$
- **Positive \times Negative = Negative** $3 \times (-4) = -12$
- **Brackets together means to multiply** $(3)(4) = 12$

Student practice:

Instructor led



1. $2 \times 3 =$

2. $-2 \times 3 =$

3. $2 \times -3 =$

4. $-2 \times 3 =$

Exercise 3.1

Solve the following numerical expressions.

1. $(-5) \times 2 =$

$5 \times 2 = 10$

$= -10$

2. $8 \times (36) =$

$8 \times 6 = -48$

3. $(-2) \times 8 =$

2×8

$= -16$

4. $2 \times (7) =$

$2 \times 7 = -14$

5. $(1)(7) =$

$1 \times 7 = +7$

6. $(6) \times 3 =$

$6 \times 3 = -18$

7. $(+7)(9) =$

$7 \times 9 = 63$

8. $6 \times (8) =$

$6 \times 8 = -48$

9. $(7)(7) =$

$7 \times 7 = -49$

10. $(7)(8) =$

$7 \times 8 = +56$

11. $(4)(+6) =$

$4 \times 6 = -24$

12. $70 \times (8) =$

$70 \times 8 = -560$

13. $(8)(8) =$

$8 \times 8 = +64$

14. $(6)(+9) =$

$6 \times 9 = -54$

15. $-30 \times |-7| =$

$-30 \times +7 = -210$

16. $(50)(8) =$

$50 \times 8 = +400$

17. $(7)(+4)(2) =$

$7 \times 4 = 28$

$28 \times 2 = +56$

18. $(-5)(-6)(-8) =$ $- \times - = + \times - = -$

$6 \times 5 = 30$

$30 \times 8 = -240$

19. $4(-8)(10) =$

$4 \times 8 = 32$

$32 \times 10 = -320$

20. $(-10)(-6)(9) =$

$6 \times 9 = 54$

$54 \times 10 = +540$

Powers and Exponents, and Square Roots of Signed Numbers

Keywords

Power	The base and exponent together; for example 4^2 , is a power and means 4×4
Base	The number the exponent is attached to; in the power 4^2 , the base is 4
Exponent	The number attached to the base that tells you how many times the base is multiplied with itself; in the power 4^2 , the exponent is 2
Square root	The opposite of squaring a number. The number in the square root sign, or <i>radical</i> , is a product of a number multiplied with itself; for example $\sqrt{16} = \sqrt{4 \times 4} = 4$ 4 is the result of $\sqrt{16}$, which means that 4 is the square root of 16

Powers

When the same factor is repeated or multiplied by itself, it is easier to write the question as a power.

Examples:

- $(2)(2)(2)(2) = 2^4$ 2 is the base, 4 is the exponent
- $(-3)(-3)(-3)(-3)(-3) = -3^5$ -3 is the base, 5 is the exponent
- $\left(\frac{3}{4}\right)\left(\frac{3}{4}\right)\left(\frac{3}{4}\right) = \left(\frac{3}{4}\right)^3$ $\frac{3}{4}$ is the base, 3 is the exponent

Note: The exponent only applies to what immediately precedes it.

- Evaluate the following:

$$(-1)^5 = (-1)(-1)(-1)(-1)(-1) = -1$$

$$(-1)^8 = (-1)(-1)(-1)(-1)(-1)(-1)(-1)(-1) = 1$$

These examples show that

- A power with a **negative base** has a **positive** value when the exponent is *even*.
- A power with a **negative base** has a **negative** value when the exponent is *odd*.

Square Root of a Number

Finding the square root of a number is the opposite operation from squaring a number.

When you square a number, you multiply the number with itself.

Finding the square root means looking for a number that when multiplied with itself will form the larger number under the square root sign.

Example 1:

Squaring a number: $5^2 = 5 \times 5 = 25$

Finding the square root of a number: $\sqrt{25} = \sqrt{5 \times 5} = 5$

Example 2:

Squaring a number: $9^2 = 9 \times 9 = 81$

Finding the square root of a number: $\sqrt{81} = \sqrt{9 \times 9} = 9$

Student practice:

Solve the following numerical expressions.

When finding the square root, you can check your answer by squaring it (that is, by multiplying the number with itself) to see if you come back to the original number.

1. $(-3)^3 =$

Instructor led



2. $\sqrt{100} =$



Exercise 3.2

Solve the following numerical expressions.

1. $8^2 = 8 \times 8 = 64$

2. $4^2 = 4 \times 4 = 16$

3. $(-5)^3 = -5 \times -5 \times -5$
 $= 5 \times 5 = 25$
 $= 25 \times 5 = 125$

odd exponent negative

4. $(7)^2 = 7 \times 7 = 49$

5. $(-2)^5 = -2 \times -2 \times -2 \times -2 \times -2$
 $= 2 \times 2 = 4$
 $= 4 \times 2 = 8$
 $= 8 \times 2 = 16$
 $= 16 \times 2 = 32$

6. $(-3)^3 = -3 \times -3 \times -3$

7. $(9)^2 = 9 \times 9 = 81$

8. $\sqrt{36} = \sqrt{6 \times 6}$
 ~~$\sqrt{6 \times 6}$~~
 $= 6$

9. $\sqrt{64} = \sqrt{8 \times 8}$
 ~~$\sqrt{8 \times 8}$~~
 $= 8$

10. $\sqrt{144} = \sqrt{12 \times 12}$
 $= 12$

11. $\sqrt{9} = \sqrt{3 \times 3}$
 $= 3$

12. $\sqrt{49} = \sqrt{7 \times 7}$
 $= 7$

Division of Signed Numbers

Division works the same as multiplication.

The **quotient** of two integers with *like signs* is **positive**.

$$(+8) \div (+2) = (+4)$$

$$(-8) \div (-2) = +4$$

The **quotient** of two integers with *unlike signs* is **negative**.

$$(-8) \div (+2) = -4$$

$$(+8) \div (-2) = -4$$

Example:

$$\frac{(+16)}{(-4)} = -4 \quad \text{Remember, the fraction bar means **divide**.$$

Summary of Division Rules

- **Positive \div Positive = Positive** $12 \div 3 = 4$
- **Negative \div Negative = Positive** $(-12) \div (-3) = 4$
- **Negative \div Positive = Negative** $(-12) \div 3 = -4$
- **Positive \div Negative = Negative** $12 \div (-3) = -4$



Student practice:

Instructor led

1. $8 \div (-2) =$

2. $\frac{-16}{4} =$

3. $-30 \div (-5) =$



Exercise 3.3

Solve the following numerical expressions.

$$1. \quad (-1) \div (+1) = -$$

$$1 \div 1 = 1$$

$$2. \quad -4 \div (-1) =$$

$$4 \div 1 = +4$$

$$3. \quad (24) \div (4) =$$

$$24 \div 4 = +6$$

$$4. \quad (-15) \div 5 =$$

$$15 \div 5 = -3$$

$$5. \quad (-18) \div 3 =$$

$$18 \div 3 = -6$$

$$6. \quad -6 \div (-2) =$$

$$6 \div 2 = +3$$

$$7. \quad (-63) \div 9 =$$

$$63 \div 9 = 7$$

$$= -7$$

$$8. \quad -12 \div (-3) =$$

$$12 \div 3 = +4$$

$$9. \quad -12 \div (-4) =$$

$$12 \div 4 = +3$$

$$10. \quad 49 \div (-7) =$$

$$49 \div 7 = -7$$

$$11. \quad -45 \div (-9) =$$

$$45 \div 9 = +5$$

$$12. \quad 72 \div (-8) =$$

$$72 \div 8 = -9$$

$$13. \quad -10 \div (-2) =$$

$$10 \div 2 = +5$$

$$14. \quad (18) \div (-6) =$$

$$18 \div 6 = -3$$

$$15. \quad -56 \div (-7) =$$

$$56 \div 7 = +8$$

$$16. \quad -1800 \div (-9) =$$

$$1800 \div 9 = +200$$

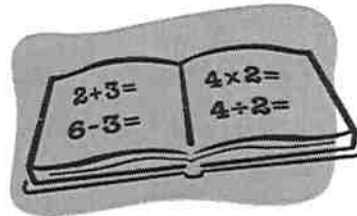
Order of Operations with Integers

Think about ...

- ◆ What is a numerical expression?
- ◆ Have you ever heard of BEDMAS?

Keywords

Arithmetic operations	Addition, subtraction, multiplication, division, etc.
BEDMAS	An acronym used to help students remember in what order to perform operations: B rackets, E xponents, D ivision and M ultiplication, A ddition, and S ubtraction
Numerical expression	A combination of numbers and one or more arithmetic operations; for example, $(4 + 5) \times 8 =$
Order of operations	The rules we follow when calculating numerical expressions



Order of Operations with Whole Numbers

A numerical expression is a combination of numbers and one or more arithmetic operation symbols. Arithmetic operations are addition, subtraction, multiplication, and division.

The following are examples of numerical expressions:

- $4 + 20 - 7$
- $(2 + 3) - 7$
- $(6 \times 2) \div 20$
- $5 \div (20 \times 3)$
- $5 \times (42 + 3)$

But what do you do when you see something like $7 + (6 \times 5) = ?$

What part should you calculate first? Should you start at the left and go to the right? Or go from right to left?

If you do the calculations in the wrong order, you will get an incorrect answer. For example, look at how two students evaluated the following numerical expression: $3 + 4 \times 2 = ?$

Student 1	Student 2
$3 + 4 \times 2 = ?$	$3 + 4 \times 2 = ?$
$= 7 \times 2$	$= 3 + 8$
$= 14$	$= 11$

It seems that each student interpreted the problem differently, resulting in two different answers. Student 1 performed the operation of addition first, then multiplication, whereas Student 2 performed multiplication first, then addition. When performing arithmetic operations, there can only be one correct answer. We need a set of rules to avoid this kind of confusion.

Mathematicians have devised a standard order of operations for calculations involving more than one arithmetic operation.

Order of Operations Rules

Rule 1: Do things in brackets first.

Example:

$$\checkmark 6 \times (5 + 3) = 6 \times 8 = 48$$

$$\times 6 \times (5 + 3) = 30 + 3 = 33 \text{ (wrong)}$$

Rule 2: Complete exponents before you multiply, divide, add, or subtract.

Example:

$$\checkmark 5 + (2)^3 = 5 + 8 = 13$$

$$\times 5 + (2)^3 = 7^3 = 343 \text{ (wrong)}$$

Rule 3: Multiply or divide (working from left to right) before you add or subtract.

Example:

$$\checkmark 2 + 5 \times 3 = 2 + 15 = 17$$

$$\times 2 + 5 \times 3 = 7 \times 3 = 21 \text{ (wrong)}$$

Otherwise, just go from left to right.

Example:

$$\checkmark 30 \div 5 \times 3 = 6 \times 3 = 18$$

$$\times 30 \div 5 \times 3 = 30 \div 15 = 2 \text{ (wrong)}$$

Rule 4: Add or subtract (working from left to right)**Example:**

$$✓ \quad 12 - 2 + 8 = 10 + 8 = 18$$

$$✗ \quad 12 - 2 + 8 = 12 - 10 = 2 \quad (\text{wrong})$$

How do you remember it all? BEDMAS!

BEDMAS is an acronym that stands for:

B **B**rackets**E** **E**xponents**DM** **D**ivision and **M**ultiplication (left to right)**AS** **A**ddition and **S**ubtraction (left to right)

It is important to understand that

- Division and multiplication rank equally (and go left to right)
- Addition and subtraction rank equally (and go left to right)

Example 1:

$$2(5 + 9) - 7 = \quad \text{Brackets first}$$

$$2(14) - 7 = \quad \text{Multiply next}$$

$$28 - 7 = 21 \quad \text{Then subtract}$$

Example 2:

$$(4 + 3) \times (6 - 3) \div 7 =$$

According to BEDMAS, we perform operations in brackets first.

$$(4 + 3) = 7$$

Then we complete the brackets left to right.

$$(6 - 3) = 3$$

Next, we multiply.

$$7 \times 3 = 21$$

And last, we divide.

$$21 \div 7 = 3$$

So, using BEDMAS,

$$(4 + 3) \times (6 - 3) \div 7 = 3$$

Student practice:

1. $7 + 3 \times 5 =$

Instructor led

2. $(7 + 3) \times 4 \div 2 - 5 \times 6 =$

3. $7 \times 2 + [7 + 3 \times (5 - 2)] - 4 \times 2 =$

Solutions:

22

-10

22

Video examples:

Examples 1 and 2



Example 3

Exercise 3.4

Find the solution to each of the following numerical expressions. Remember to use the order of operations rules (BEDMAS).

$$1. \quad 5 + \underline{3 \times 2} = \quad 3 \times 2 = 6$$

$$5 + 6 = 11$$

$$2. \quad 12 - \underline{6 \div 3} = \quad 6 \div 3 = 2$$

$$12 - 2 = 10$$

$$3. \quad \underline{(24 - 4)} \div 5 = \quad 24 - 4 = 20$$

$$20 \div 5 = 4$$

$$4. \quad 6 + \underline{(9 - 2)^2} = \quad 9 - 2 = 7$$

$$6 + \underline{7^2} \quad 7 \times 7 = 49$$

$$6 + 49 = 55$$

$$5. \quad \underline{(22 - 18)} \times \underline{(12 - 6)} = \quad 22 - 18 = 4 \quad 12 - 6 = 6$$

$$4 \times 6 = 24$$

$$6. \quad \underline{(24 + 8)} \div \underline{(14 - 10)} = \quad 24 + 8 = 32 \quad 14 - 10 = 4$$

$$32 \div 4 = 8$$

$$7. \quad 67 + \underline{(8 - 7)} \times 3 + 5 = \quad 8 - 7 = 1$$

$$= 67 + \underline{1} \times 3 + 5 \quad 1 \times 3 = 3$$

$$= 67 + 3 + 5 \quad 67 + 3 = 70$$

$$= 70 + 5 = 75$$

$$8. \quad \underline{(34 - 14)} + 16 \div 2 \underline{(10 - 5)} = \quad 34 - 14 = 20 \quad 10 - 5 = 5$$

$$20 + 16 \div 2(5) \quad 16 \div 2 = 8$$

$$20 + 8 \times 5 \quad 8 \times 5 = 40$$

$$20 + 40 = 60$$

$$\begin{array}{l}
 9. \quad 67 - 2(17 + 8) - 10 + 2 \times 4 = \quad 17 + 8 = 25 \\
 \quad \quad 67 - 2 \times 25 - 10 + 2 \times 4 \quad 2 \times 25 = 50 \quad 2 \times 4 = 8 \\
 \quad \quad 67 - 50 - 10 + 8 \quad 67 - 50 = 17 \\
 \quad \quad \quad 17 - 10 + 8 \quad 17 - 10 = 7 \\
 \quad \quad \quad \quad 7 + 8 = 15 \\
 10. \quad (5 \times 7) + (12 - 6) = \quad 5 \times 7 = 35 \quad 12 - 6 = 6 \\
 \quad \quad 35 + 6 = 41
 \end{array}$$

$$\begin{array}{l}
 11. \quad 17 + 12 - 8(12 - 9) = \quad 12 - 9 = 3 \\
 \quad \quad 17 + 12 - 8 \times 3 \quad 8 \times 3 = 24 \\
 \quad \quad 17 + 12 - 24 \quad 17 + 12 = 29 \\
 \quad \quad \quad 29 - 24 = 5
 \end{array}$$

$$\begin{array}{l}
 12. \quad (18 - 8) \times (20 - 10) = \quad 18 - 8 = 10 \quad 20 - 10 = 10 \\
 \quad \quad 10 \times 10 = 100
 \end{array}$$

$$\begin{array}{l}
 13. \quad 12 + (2 \times 8) - 18 \div 2 + 13 = \quad 2 \times 8 = 16 \quad 18 \div 2 = 9 \\
 \quad \quad 12 + 16 - 9 + 13 \quad 12 + 16 = 28 \\
 \quad \quad \quad 28 - 9 + 13 \quad 28 - 9 = 19 \\
 \quad \quad \quad \quad 19 + 13 = 31
 \end{array}$$

$$\begin{array}{l}
 14. \quad 64 \div (17 - 9) + 3(8 + 5) - 7 = \quad 17 - 9 = 8 \quad 8 + 5 = 13 \\
 \quad \quad 64 \div 8 + 3 \times 13 - 7 \quad 64 \div 8 = 8 \quad 3 \times 13 = 39 \\
 \quad \quad \quad 8 + 39 - 7 \quad 8 + 39 = 47 \\
 \quad \quad \quad \quad 47 - 7 = 40
 \end{array}$$

Order of Operations with Integers

Examples

Integers:

$$\begin{aligned} -12 \div (8 - 5) - 10 &= && \text{Brackets first} \\ -12 \div 3 - 10 &= && \text{Division next} \\ -4 - 10 &= -14 && \text{Then add or subtract} \end{aligned}$$

Exponents:

$$\begin{aligned} (-5)^2 - (4 - 7)^2 - 5 &= && \text{Brackets first} \\ (-5)^2 - (-3)^2 - 5 &= && \text{Exponents next} \\ 25 - 9 - 5 &= 11 && \text{Then add or subtract} \end{aligned}$$

Absolute value brackets:

$$\begin{aligned} (-5)^2 |9 - 11| + (-3) &= && \text{Brackets first} \\ (-5)^2 |-2| + (-3) &= && \text{Remember, the absolute value of } -2 \text{ is } +2 \\ (-5)^2 \times 2 + (-3) &= && \text{Exponents next} \\ 25 \times 2 + (-3) &= && \text{Then multiply} \\ 50 + (-3) &= 47 && \text{Finally, add or subtract} \end{aligned}$$

Student practice:

1. $-1[(-7) + 2(3 + 2)] - (5)^2$

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Solution and video:

-28



2. $12 + (5 - 1) \times 3^2 - 8 \div \sqrt{4} =$

44



Exercise 3.5

Solve the following numerical expressions.

1. $5 - 5^2 =$ $5 \times 5 = 25$

$$\begin{array}{r} 5 - 25 \\ + 5 + -25 \\ \hline -20 \end{array}$$

$25 - 5 = 20$

2. $2 - (-5) + 3^2 =$ $3 \times 3 = 9$

$$\begin{array}{r} 2 + +5 + 9 \\ \hline 7 + 9 = 16 \end{array}$$

$2 + 5 = 7$

3. $4(3^2) + 7(3+9) - (-6) =$ $3 \times 3 = 9$ $3 + 9 = 12$

$$\begin{array}{r} 4 \times 9 + 7 \times 12 + +6 \\ \hline 36 + 84 + 6 \end{array}$$

$4 \times 9 = 36$

$7 \times 12 = 84$

$36 + 84 + 6$

$36 + 84 = 120$

$120 + 6 = 126$

4. $6 \times \sqrt{25} - 4 \times \sqrt{16} =$

$$\begin{array}{r} 6 \times 5 - 4 \times 4 \\ \hline 30 - 16 = 14 \end{array}$$

$\sqrt{5 \times 5} = 5$

$6 \times 5 = 30$

$\sqrt{4 \times 4} = 4$

$4 \times 4 = 16$

$30 - 16 = 14$

5. $-17 + 8 \times \sqrt{9} - (11-5)^2 =$

$$\begin{array}{r} -17 + 8 \times 3 - 6^2 \\ \hline -17 + 24 - 36 \end{array}$$

$$\begin{array}{r} -17 + 24 - 36 \\ \hline -53 + 24 = -29 \end{array}$$

$$\begin{array}{r} -17 + 24 - 36 \\ \hline -53 + 24 = -29 \end{array}$$

$\sqrt{3 \times 3} = 3$

$6 \times 6 = 36$

$8 \times 3 = 24$

$-17 + -36 = -53$

$53 - 24 = 29$

$11 - 5 = 6$

6. $|-12| \div 4 + 2 \times |(-2)^3| \div 4 =$

$$\begin{array}{r} | -12 | \div 4 + 2 \times | -8 | \div 4 \\ \hline 12 \div 4 + 2 \times 8 \div 4 \end{array}$$

$$\begin{array}{r} 12 \div 4 + 2 \times 8 \div 4 \\ \hline 3 + 16 \div 4 \end{array}$$

$$\begin{array}{r} 3 + 16 \div 4 \\ \hline 3 + 4 = 7 \end{array}$$

$3 + 4 = 7$

$-2 \times -2 \times -2 = -8$

$12 \div 4 = 3$

$2 \times 8 = 16$

$16 \div 4 = 4$

Problem Solving

Tips for Solving Application Problems

Read the tips below on solving application problems:

- **Step 1:** Read the question carefully. Read the problem several times.
- **Step 2:** Work out a plan. Write an equation to solve the problem.
- **Step 3:** Estimate. Is your answer reasonable?
- **Step 4:** Solve the problem. Is your answer reasonable?
- **Step 5:** Write a statement answering the problem.

Words that can be used to identify operations:

Addition	Subtraction	Multiplication	Division	Equals
sum	difference	product	quotient	is
total	minus	times	divide	is the same as
increased by	less than	double	per	equals
plus	more than	triple	divided equally	equal to
added to	decrease	of	divided by	results in
more	loss	twice		
gain	fewer			

Exercise 3.6

Solve the following problems. Write your answers in sentence form.

1. A salesman takes a business trip and travels the following number of kilometres (km) each day: on Monday he travels 385 km, on Tuesday 412 km, on Wednesday 394 km, on Thursday 403 km, and on Friday 431 km. What is the total number of kilometres that he travelled that week?

$$385 + 412 + 394 + 403 + 431 = 2025$$

2025 km

$$\begin{array}{r} 385 \\ 412 \\ 394 \\ 403 \\ +431 \\ \hline 2025 \end{array}$$

2. Mary has volunteered to bake desserts for her school bake sale. She baked two dozen (24) cupcakes and plans to sell them for \$3.25 each. She also baked a dozen (12) brownies and will sell them for \$2.50 each. How much money will Mary raise if she sells all of her baked goods?

$$3.25 \times 24 + 2.5 \times 12$$

$$78 + 30$$

$$108$$

\$108

3. In June, the Anderson family made four deposits of \$1 782 each to their bank account. They also withdrew \$5 931 for expenses. What was their account balance for the month of June, assuming they had no money in the account at the start of the month?

$$4(1782) - 5931$$

$$= 7182 - 5931$$

$$= \$1197 \text{ at the end of June}$$

4. A group of six students is planning a summer trip to Germany. The total cost of all six flights is \$8 400. Each person will pay \$600 for hotel fees. If the six students all pay equally for the flights, what is the cost of the entire trip for each student, including the hotel fees?

$$8400 \div 6 + 600$$

$$= 1400 + 600 = \$2000 \text{ each}$$

5. On Sunday, June 21, the temperature was 27°C. The temperature dropped 2°C per day over the next seven days. What was the temperature on June 28?

$$27 - 2(7)$$

$$27 - 14$$

$$13^{\circ}\text{C on June 28}$$

Unit 4: Operations with Signed Decimals

Adding and Subtracting Decimals

Adding Signed Decimals

If the signs are the same, add the numbers and the sign remains the same.

Example 1: $-9.087 + -15.31$

9.087	Step 1: Line up the decimals and add zeros if necessary
$+ 15.310$	Step 2: The signs are the same (both negative), so add the decimals and keep the negative sign. Since both signs are negative, the answer is negative.
-24.397	

If the signs are opposite, use the absolute value of each number. Then subtract the smaller absolute value from the larger absolute number. The sign of the larger number will be the sign of your answer.

Example 2: $(+9.087) + (-15.31)$

15.310	Step 1: Line up the decimals again, putting the largest absolute value on top. 15 is larger than 9, so it needs to go first.
$- 9.087$	
-6.223	Step 2: Subtract the numbers. Since the larger number (-15.31) is negative, the answer is also negative.

Subtracting Signed Decimals

When subtracting signed decimals, change the subtraction sign to addition, and change the sign of the number behind it to its opposite. Then follow the rules for adding signed decimals.

Example: $(-12.2) - (-15.31)$

$(-12.2) + (+15.31)$	Step 1: Change the subtraction sign to addition and change the sign of the number behind it to its opposite
15.91	Step 2: Since the signs are opposite, subtract the absolute values. The largest number is positive, so the answer will be positive.
$- 12.20$	
$+3.71$	

Exercise 4.1

Solve the following numerical expressions.

1. $(24.008) + (-0.995) =$

$$\begin{array}{r} 24.008 \\ - 0.995 \\ \hline 23.013 \end{array}$$

2. $(-6.05) + (-39.7) =$

$$\begin{array}{r} 39.70 \\ + 6.05 \\ \hline 45.75 \end{array} \quad -45.75$$

3. $0.9 - 7.29 =$

$$\begin{array}{r} +0.9 \\ + -7.29 \\ \hline -6.39 \end{array}$$

4. $(-2) - 4.99 =$

$$\begin{array}{r} -2 \\ + -4.99 \\ \hline -6.99 \end{array}$$

5. $-5.009 + 0.73 =$

$$\begin{array}{r} 5.009 \\ - 0.730 \\ \hline 4.279 \end{array} \quad -4.279$$

6. $-1.7035 - (-6.7) =$

$$\begin{array}{r} -1.7035 \\ + 6.7 \\ \hline 4.9965 \end{array} \quad +4.9965$$

7. $8000 + (-8002.63) - 8 =$

$$\begin{aligned} &= 8000 + -8002.63 + -8 \\ &= 8000 + -8010.63 \\ &= -10.63 \end{aligned}$$

8. $-6.5 + 0.7 =$

$$\begin{array}{r} 6.5 \\ - 0.7 \\ \hline 5.8 \end{array} \quad -5.8$$

9. $-42.671 - 194.9 =$

$$\begin{array}{r} -42.671 \\ + -194.9 \\ \hline -237.571 \end{array}$$

10. $-8.4 + (-50.83) =$

$$\begin{array}{r} 50.83 \\ + 8.4 \\ \hline 59.23 \end{array} \quad -59.23$$

11. $0.77 - 3.06 =$

$$\begin{array}{r} 3.06 \\ - 0.77 \\ \hline 2.29 \end{array} \quad -2.29$$

12. $-6.409 + 8.224 =$

$$\begin{array}{r} 8.224 \\ - 6.409 \\ \hline 1.815 \end{array}$$

13. $-489.7 - 38 =$

$$\begin{array}{r} 489.7 \\ + 38.0 \\ \hline 527.7 \end{array} \quad -527.7$$

14. $(+11.3) - (-25.652) - (+13.91) =$

$$\begin{aligned} &= +11.3 + +25.652 + -13.91 \\ &\begin{array}{r} 11.300 \\ + 25.652 \\ \hline 36.952 \end{array} \quad 36.952 - 13.91 \\ &= +23.042 \end{aligned}$$

$$\begin{array}{l}
 15. \quad 12.2 - (+15.2) + (-34.7) - (-12.8) = \\
 \quad 12.2 + -15.2 + -34.7 + +12.8 \\
 +12.2 + +12.8 + -15.2 + -34.7 \\
 \quad 25.0 \quad + -49.9 \\
 \quad 49.9 - 25 = -24.9
 \end{array}$$

$$\begin{array}{l}
 16. \quad (-1.3) - (+0.5) - (-3.8) - (-9.7) = \\
 \quad -1.3 + -0.5 + +3.8 + +9.7 \\
 \quad -1.8 + +12.5 \\
 \quad 12.5 - 1.8 \\
 \quad +10.7
 \end{array}$$

Multiplying and Dividing Signed Decimals

Multiplication Rules of Signs

- Positive \times Positive = Positive
- Negative \times Negative = Positive
- Negative \times Positive = Negative
- Positive \times Negative = Negative
- Brackets together means to multiply



Example: -0.008×0.06

$$\begin{array}{r}
 -0.008 \quad \text{Apply the rule Negative} \times \text{Positive} = \text{Negative} \\
 \times 0.06 \quad \text{Multiply as normal and add the negative sign to the result.} \\
 \hline
 -0.00048
 \end{array}$$

Division Rules of Signs

- Positive \div Positive = Positive
- Negative \div Negative = Positive
- Negative \div Positive = Negative
- Positive \div Negative = Negative

Example: $0.3 \overline{) -0.0036}$

$$\begin{array}{r}
 3 \overline{) -0.036} \quad \text{Move the decimal in the first number once to the right and do the same for} \\
 \quad \text{the second number.}
 \end{array}$$

$$\begin{array}{r}
 -0.006 \\
 3 \overline{) -0.036} \quad \text{Apply the rule Negative} \div \text{Positive} = \text{Negative} \text{ and put a negative sign in the} \\
 \quad \text{answer.}
 \end{array}$$

$$\begin{array}{r}
 \quad 36 \\
 \hline
 \quad 0 \\
 \hline
 \quad 0
 \end{array}
 \quad \text{Now divide as normal.}$$

Exercise 4.2

Solve the following numerical expressions.

$$1. \quad (-0.72) \times (-0.06) = \text{---} \times \text{---} = \text{+}$$

$$\begin{array}{r} 0.72 \\ \times 0.06 \\ \hline 0.0432 \end{array} = 0.0432$$

$$2. \quad 2.3 \times (-0.8) =$$

$$\begin{array}{r} 2.3 \\ \times 0.8 \\ \hline -1.84 \end{array}$$

$$3. \quad (-0.1)(-3.5) =$$

$$\begin{array}{r} 3.5 \\ \times 0.1 \\ \hline 0.35 \end{array} = 0.35$$

$$4. \quad (-0.6) \times 0.3 =$$

$$\begin{array}{r} 0.6 \\ \times 0.3 \\ \hline 0.18 \end{array} = -0.18$$

$$5. \quad 0.42(-1.9) =$$

$$\begin{array}{r} 0.42 \\ \times 1.9 \\ \hline 378 \\ 420 \\ \hline 0.798 \end{array} = -0.798$$

$$6. \quad (-0.6) \times (-0.8) =$$

$$\begin{array}{r} 0.6 \\ \times 0.8 \\ \hline 0.48 \end{array} = 0.48$$

$$7. \quad (-1.2)(6.8) =$$

$$\begin{array}{r} 6.8 \\ \times 1.2 \\ \hline 136 \\ 680 \\ \hline 8.16 \end{array} = -8.16$$

$$8. \quad (-4.37)(-2.08) =$$

$$\begin{array}{r} 4.37 \\ \times 2.08 \\ \hline 3496 \\ 87400 \\ \hline 9.0896 \end{array}$$

$$9. \quad 27.3 \div (-7) =$$

$$\begin{array}{r} 3.9 \\ 7 \overline{) 27.3} \\ \underline{-21} \\ 63 \\ \underline{-63} \\ 0 \end{array} = -3.9$$

$$10. \quad -4.23 \div -9 =$$

$$\begin{array}{r} 0.47 \\ 9 \overline{) 4.23} \\ \underline{36} \\ 63 \\ \underline{-63} \\ 0 \end{array} = 0.47$$

11. $(-20.01) \div (-0.5) =$

$$\begin{array}{r} 0.5 \overline{) 20.01} \\ \underline{10.00} \\ 10.01 \\ \underline{5.00} \\ 5.01 \\ \underline{5.00} \\ 0.01 \\ \underline{0.00} \\ 0.01 \\ \underline{0.00} \\ 0.01 \\ \underline{0.00} \\ 0.01 \\ \underline{0.00} \\ 0.01 \\ \underline{0.00} \\ 0.01 \\ \underline{0.00} \\ 0.01 \end{array} \quad 40.02$$

13. $(-0.108) \div 1.8 =$

$$\begin{array}{r} 1.8 \overline{) 0.108} \\ \underline{0.06} \\ 1.08 \\ \underline{1.08} \\ 0 \end{array} \quad -0.06$$

15. $(-1.7) \div 0.09 =$

$$\begin{array}{r} 0.09 \overline{) 1.70} \\ \underline{0.081} \\ 0.089 \\ \underline{0.080} \\ 0.009 \\ \underline{0.008} \\ 0.001 \\ \underline{0.000} \\ 0.001 \\ \underline{0.000} \\ 0.001 \\ \underline{0.000} \\ 0.001 \end{array} \quad -18.9$$

17. $(-27.3) \div 100 =$

$$\begin{array}{r} 27.3 \div 100 \\ -0.273 \end{array}$$

12. $54 \div -1.5 =$

$$\begin{array}{r} 1.5 \overline{) 54.0} \\ \underline{30} \\ 24.0 \\ \underline{15} \\ 9.0 \\ \underline{9.0} \\ 0 \end{array} \quad -36$$

14. $-3.1 \div -0.006 =$

$$\begin{array}{r} 0.006 \overline{) 3.108} \\ \underline{0.006} \\ 0.000 \end{array} \quad 516.7$$

$$\begin{array}{r} 6 \overline{) 3100.0} \\ \underline{30} \\ 10 \\ \underline{6} \\ 40 \\ \underline{36} \\ 40 \\ \underline{36} \\ 4 \end{array}$$

16. $-240.8 \div 9 =$

$$\begin{array}{r} 9 \overline{) 240.80} \\ \underline{18} \\ 60 \\ \underline{54} \\ 68 \\ \underline{63} \\ 50 \end{array} \quad -26.76$$

18. $(-2) \div (-20) =$

$$\begin{array}{r} 20 \overline{) 2.00} \\ \underline{20} \\ 0 \end{array} \quad 0.1$$

Order of Operations with Decimals

Example:

$$\begin{aligned} -0.75 + 0.8(-0.6 + 0.2) &= && \text{Brackets first} \\ -0.75 + 0.8(-0.4) &= && \text{Multiply next} \\ 0.75 - 0.32 &= -1.07 && \text{Then add or subtract} \end{aligned}$$

Student practice:

$$(-0.19 + 2.4) - 3(-0.04 - 0.02)$$

Instructor led



Answer: 2.53

Exercise 4.3

Find the solutions to the following numerical expressions. Remember to use the order of operations rules (BEDMAS).

$$\begin{aligned} 1. \quad -8.9 + 4^2 \div (-0.02) &= \\ -8.9 + 16 \div -0.02 & \\ -8.9 + -800 & \\ -808.9 & \end{aligned}$$

$$\begin{aligned} 2. \quad \frac{-0.18 + 2.5}{2.32} + \frac{4(0.05)}{0.2} &= \\ 2.32 + 0.2 & \\ 2.52 & \end{aligned}$$

$$\begin{aligned} 3. \quad \frac{-0.7(-3)}{2.1} - \frac{-0.04}{2} &= \\ 2.1 - -0.02 & \\ 2.1 + 0.02 & \\ 2.12 & \end{aligned}$$

4. Gino had $-\$15.46$ in his bank account at the end of June. During the month of July, he made deposits of $\$75.38$ and $\$182.25$. He made two withdrawals of $\$98.99$ and $\$159.26$. How much money is in his bank account at the end of July?

$$\begin{aligned} -15.46 + 75.38 + 182.25 - 98.99 - 159.26 & \\ 75.38 + 182.25 + -15.46 + -98.99 + -159.26 & \\ + 257.63 + -273.71 & \\ 273.71 - 257.63 & = -16.08 \end{aligned}$$

Unit 5: Operations with Signed Fractions

The rules don't change for operations with fractions. All that changes for signed fractions is how the signs are affected. The same rules apply as for integers.

Adding and Subtracting Signed Fractions

Adding Signed Fractions

If the signs are the same, add the fractions and the sign remains the same.

- Remember to find the lowest common denominator first, if necessary.

Example 1:

$$-\frac{3}{4} + \left(-\frac{5}{6}\right) =$$

Step 1: Find the lowest common denominator. For 4 and 6, it is 12.

$$-\frac{3 \times 3}{4 \times 3} + \left(-\frac{5}{6}\right) \times \frac{2}{2} =$$

Step 2: Multiply the numerator and denominator by the same number to make both denominators 12.

$$-\frac{9}{12} + \left(-\frac{10}{12}\right) = -\frac{19}{12}$$

Step 3: Rewrite the fractions with the common denominator and add the numerators.

$$= -1\frac{7}{12}$$

Step 4: Change the fraction to a mixed number.

If the signs are opposite, use the absolute values of the numbers. Subtract the smaller absolute value from the larger absolute value. The sign of the larger number will be the sign of your answer.

Example 2:

$$\frac{5}{12} + \left(-\frac{5}{9}\right) =$$

To find the common denominator, use the product of prime factors:

- 12 is $(2 \times 2 \times 3)$ and 9 is (3×3)
- The common denominator is $(2 \times 2) \times (3 \times 3) = 36$

$$\frac{15}{36} + \left(-\frac{20}{36}\right) = \left(-\frac{5}{36}\right)$$

The signs are opposite, so subtract the absolute values:
 $20 - 15 = 5$

The largest number is negative, so the answer will be negative.

Subtracting Signed Fractions

Example:

When subtracting signed fractions, change the minus sign to addition and change the sign of the number behind it to its opposite. Then follow the rules for adding signed fractions.

$$-2\frac{1}{2} - \left(+3\frac{3}{4}\right) =$$

Find a common denominator and change subtraction to addition.

$$-2\frac{2}{4} + \left(-3\frac{3}{4}\right) = -5\frac{5}{4}$$

The signs are the same (both negative), so the answer will be negative. Add the whole numbers, then add the fractions.

$$= -6\frac{1}{4}$$

Finally, reduce to lowest terms.

Student practice:

1. $-\frac{3}{4} - \frac{7}{6} - \frac{3}{6} =$

Instructor led



2. $3\frac{1}{8} + \frac{3}{4} + \left(-2\frac{1}{6}\right) =$



Exercise 5.1

Solve the following numerical expressions.

$$1. \left(-\frac{1}{14}\right) + \left(-\frac{3}{7}\right) =$$

$$-\frac{1}{14} + -\frac{6}{14} = -\frac{7}{14}$$

$$= -\frac{1}{2}$$

$$3. \frac{3}{4} + \frac{1}{8} =$$

$$\frac{6}{8} + \frac{1}{8} = \frac{7}{8}$$

$$5. -\frac{7}{24} - \frac{3}{8} =$$

$$-\frac{7}{24} + -\frac{9}{24}$$

$$= -\frac{16}{24} = -\frac{2}{3}$$

$$7. \left(-\frac{7}{9}\right) - \frac{5}{6} =$$

$$-\frac{7 \times 2}{9 \times 2} + -\frac{5 \times 3}{6 \times 3}$$

$$-\frac{14}{18} + -\frac{15}{18} = -\frac{29}{18} = -1\frac{11}{18}$$

$$9. \left(+\frac{8}{20}\right) - \left(+\frac{4}{5}\right) =$$

$$\frac{8}{20} + -\frac{4 \times 4}{5 \times 4}$$

$$\frac{8}{20} + -\frac{16}{20} = -\frac{8}{20} = -\frac{2}{5}$$

$$11. -2\frac{1}{2} + \left(-3\frac{4}{5}\right) =$$

$$-2\frac{5}{10} + -3\frac{8}{10}$$

$$= -5\frac{13}{10} = -6\frac{3}{10}$$

$$2. \left(-\frac{2}{9}\right) + \frac{2}{3} =$$

$$-\frac{2}{9} + \frac{6}{9} = \frac{6}{9} - \frac{2}{9} = \frac{4}{9}$$

$$4. \left(-\frac{5}{8}\right) - \left(-\frac{1}{12}\right) =$$

$$-\frac{5}{8} + +\frac{1}{12} = -\frac{5 \times 3}{8 \times 3} + +\frac{1 \times 2}{12 \times 2}$$

$$= -\frac{15}{24} + \frac{2}{24} = -\frac{13}{24}$$

$$6. -2 + \frac{3}{5} =$$

$$= 2 - \frac{3}{5} = 1\frac{2}{5} = -1\frac{2}{5}$$

$$8. \left(-\frac{7}{9}\right) - \left(-\frac{3}{6}\right) = -\frac{7 \times 2}{9 \times 2} + \frac{3 \times 3}{6 \times 3}$$

$$= -\frac{14}{18} + \frac{9}{18} = -\frac{5}{18}$$

$$10. \left(-\frac{3}{4}\right) + \frac{5}{8} =$$

$$-\frac{6}{8} + \frac{5}{8} = -\frac{1}{8}$$

$$12. 5\frac{1}{3} - 8\frac{1}{4} = 5\frac{1 \times 4}{3 \times 4} + -8\frac{1 \times 3}{4 \times 3}$$

$$= 5\frac{4}{12} + -8\frac{3}{12}$$

$$= 8\frac{3}{12} - 5\frac{4}{12} = 7\frac{15}{12} - 5\frac{4}{12}$$

$$= 2\frac{11}{12} = -2\frac{11}{12}$$

$$13. -16\frac{3 \times 3}{7 \times 3} + \left(-27\frac{2}{3}\right) \times 7 =$$

$$-16\frac{9}{21} + -27\frac{14}{21}$$

$$-43\frac{23}{21} = -44\frac{2}{21}$$

$$14. 12\frac{5 \times 7}{8 \times 7} + \left(-15\frac{2}{7}\right) \times 8 =$$

$$12\frac{35}{56} + -15\frac{16}{56} = 15\frac{16}{56} - 12\frac{35}{56}$$

$$= 14\frac{56+16}{56} - 12\frac{35}{56} = 14\frac{72}{56} - 12\frac{35}{56}$$

$$= -2\frac{37}{56}$$

$$15. -6\frac{1 \times 3}{2 \times 3} + 3\frac{2 \times 2}{3 \times 2} + \left(-7\frac{5}{6}\right) =$$

$$16. -6\frac{1 \times 2}{5 \times 2} - 3\frac{2 \times 2}{5 \times 2} - 4\frac{9}{10} =$$

$$= -6\frac{2}{10} + -3\frac{4}{10} + -4\frac{9}{10}$$

$$= -13\frac{15}{10} = -14\frac{5}{10} = -14\frac{1}{2}$$

$$= -6\frac{3}{6} + 3\frac{4}{6} + -7\frac{5}{6}$$

$$= -6\frac{3}{6} + -7\frac{5}{6} + +3\frac{4}{6}$$

$$= -13\frac{8}{6} + 3\frac{4}{6} = -10\frac{4}{6} = -10\frac{2}{3}$$

$$17. -3\frac{1 \times 6}{3 \times 6} - 15\frac{5 \times 2}{9 \times 2} - 13\frac{11}{18} =$$

$$18. 17\frac{1 \times 3}{8 \times 3} + 13\frac{2 \times 8}{3 \times 8} + \left(-22\frac{5}{6}\right) \times 4 =$$

$$= 17\frac{3}{24} + 13\frac{16}{24} + -22\frac{20}{24}$$

$$= 30\frac{19}{24} + -22\frac{20}{24}$$

$$= 29\frac{24}{24} - 22\frac{20}{24} = 7\frac{23}{24}$$

$$19. \frac{1}{3} + \left(-\frac{3}{5}\right) + \frac{7}{15} - \frac{5}{30} + \left(-\frac{7}{10}\right) = \text{LCD} = 30$$

$$\frac{1 \times 10}{3 \times 10} + -\frac{3 \times 6}{5 \times 6} + \frac{7 \times 2}{15 \times 2} + -\frac{5}{30} + -\frac{7 \times 3}{10 \times 3}$$

$$= \frac{10}{30} + \frac{14}{30} + -\frac{18}{30} + -\frac{5}{30} + -\frac{21}{30} = +\frac{24}{30} + -\frac{44}{30} = \frac{44-24}{30} = \frac{20}{30} = \frac{2}{3}$$

$$20. 2\frac{1}{2} + \left(-4\frac{9}{16}\right) + 8\frac{1}{4} - 7\frac{5}{8} = \text{LCD} = 16$$

$$= 2\frac{1 \times 8}{2 \times 8} + 8\frac{1 \times 4}{4 \times 4} + -4\frac{9}{16} + -7\frac{5 \times 2}{8 \times 2}$$

$$= 2\frac{8}{16} + 8\frac{4}{16} + -4\frac{9}{16} + -7\frac{10}{16}$$

$$= 10\frac{12}{16} + -11\frac{19}{16}$$

$$= 11\frac{19}{16} - 10\frac{12}{16}$$

$$= 1\frac{7}{16}$$

$$= -1\frac{7}{16}$$

Multiplying and Dividing Signed Fractions

Multiplying Signed Fractions

Multiplying Rules of Signs

- **Positive** × **Positive** = **Positive**
- **Negative** × **Negative** = **Positive**
- **Negative** × **Positive** = **Negative**
- **Positive** × **Negative** = **Negative**
- **Brackets together means to multiply**



Remember the steps for multiplying fractions:

- **Step 1:** Change whole numbers or mixed numbers to improper fractions.
- **Step 2:** Reduce any number from the numerator with any number from the denominator.
- **Step 3:** Multiply the numerators, then multiply the denominators.
- **Step 4:** Change an improper fraction to a mixed fraction or whole number.

Example:

$$\begin{aligned} 3 \times \left(-\frac{2}{3}\right) &= \frac{3}{1} \times \left(-\frac{2}{3}\right) \\ &= \frac{\cancel{3}^1}{1} \times \left(-\frac{2}{\cancel{3}_1}\right) \\ &= -\frac{2}{1} = -2 \end{aligned}$$

Student practice:

1. $-7 \times \frac{3}{49} =$

Instructor led



2. $-\frac{5}{9} \times \left(-\frac{3}{15}\right) =$

Dividing Signed Fractions

Division Rules of Signs

- **Positive \div Positive = Positive**
- **Negative \div Negative = Positive**
- **Negative \div Positive = Negative**
- **Positive \div Negative = Negative**

Remember the steps for dividing fractions:

- **Step 1:** Change whole numbers or mixed numbers to improper fractions.
- **Step 2:** Never divide—always multiply by the reciprocal.
- **Step 3:** Reduce any number from the numerator with any number from the denominator.
- **Step 4:** Multiply the numerators, then multiply the denominators.
- **Step 5:** Change an improper fraction to a mixed fraction or whole number.

Example:

$$\begin{aligned} -3 \div \frac{2}{3} &= -\frac{3}{1} \div \frac{2}{3} \\ &= -\frac{3}{1} \times \frac{3}{2} \\ &= -\frac{3 \times 3}{1 \times 2} = -\frac{9}{2} = -4 \frac{1}{2} \end{aligned}$$

Student practice:

1. $-\frac{5}{6} \div \frac{3}{4} =$

Instructor led



2. $-4 \div \left(-\frac{1}{2}\right) =$

Exercise 5.2

Solve the following numerical expressions.

$$1. \quad -\frac{3}{8} \times \left(-\frac{12}{5}\right) =$$

$$= -\frac{3}{8} \times -\frac{12}{5}$$

$$= \frac{9}{10}$$

$$3. \quad -\frac{4}{9} \times \frac{3}{8} =$$

$$= -\frac{4}{9} \times \frac{3}{8}$$

$$= -\frac{1}{6}$$

$$5. \quad \left(-\frac{6}{11}\right) \left(-\frac{22}{15}\right) =$$

$$= -\frac{6}{11} \times -\frac{22}{15}$$

$$= \frac{2}{5}$$

$$7. \quad 10 \times \left(-\frac{3}{5}\right) =$$

$$= \frac{10}{1} \times -\frac{3}{5}$$

$$= -\frac{6}{1} = -6$$

$$9. \quad -20 \times \frac{4}{5} =$$

$$= -\frac{20}{1} \times \frac{4}{5}$$

$$= -\frac{16}{1} = -16$$

$$11. \quad \frac{4}{3} \div \left(-\frac{8}{9}\right) =$$

$$= \frac{4}{3} \times -\frac{9}{8}$$

$$= -\frac{3}{2} = -1\frac{1}{2}$$

$$2. \quad \frac{2}{3} \times \left(-\frac{6}{7}\right) =$$

$$\frac{2}{3} \times -\frac{6}{7} = -\frac{4}{7}$$

$$4. \quad \left(-\frac{21}{30}\right) \left(\frac{5}{7}\right) =$$

$$= -\frac{21}{30} \times \frac{5}{7} = -\frac{3}{6} = -\frac{1}{2}$$

$$6. \quad -\frac{9}{14} \times \left(-\frac{7}{72}\right) =$$

$$= -\frac{9}{14} \times -\frac{7}{72} = \frac{7}{16}$$

$$8. \quad -27 \times \left(-\frac{3}{15}\right) =$$

$$= -\frac{27}{1} \times -\frac{3}{15}$$

$$= \frac{27}{5} = 5\frac{2}{5}$$

$$10. \quad 21 \times \left(-1\frac{5}{7}\right) =$$

$$= \frac{21}{1} \times -\frac{12}{7}$$

$$= -\frac{36}{1} = -36$$

$$12. \quad -\frac{8}{15} \div \left(-\frac{4}{25}\right) =$$

$$= -\frac{8}{15} \times -\frac{25}{4}$$

$$= \frac{10}{3} = 3\frac{1}{3}$$

$$13. \frac{5}{9} \div \left(-\frac{10}{27}\right) =$$

$$= \frac{\cancel{5}^1}{9_1} \times -\frac{\cancel{27}^3}{\cancel{10}_2} = -\frac{3}{2} = -1\frac{1}{2}$$

$$15. -\frac{5}{6} \div (-10) =$$

$$= -\frac{\cancel{5}^1}{6_1} \times -\frac{1}{\cancel{10}_2} = \frac{1}{12}$$

$$17. -\frac{9}{16} \div \frac{18}{24} \div \left(-\frac{1}{2}\right) =$$

$$= -\frac{\cancel{9}^1}{\cancel{16}_8} \times \frac{\cancel{24}^2}{\cancel{18}_2} \times -\frac{\cancel{2}^1}{1} = \frac{24}{16} = \frac{3}{2} = 1\frac{1}{2}$$

$$19. \left(\frac{4}{11}\right)\left(-\frac{22}{25}\right)\left(\frac{20}{11}\right)\left(\frac{5}{6}\right)\left(-\frac{9}{8}\right) =$$

$$= \frac{\cancel{4}^1}{11_1} \times -\frac{\cancel{22}^2}{\cancel{25}_5} \times \frac{\cancel{20}^2}{11_1} \times \frac{\cancel{5}^1}{\cancel{6}_2} \times -\frac{\cancel{9}^3}{\cancel{8}_4} =$$

$$= \frac{2 \times 2 \times 3}{11} = \frac{12}{11} = 1\frac{1}{11}$$

$$14. -\frac{7}{10} \div \frac{14}{1} =$$

$$= -\frac{\cancel{7}^1}{10_1} \times \frac{1}{\cancel{14}_2} = -\frac{1}{20}$$

$$16. 4\frac{1}{2} \div (-18) =$$

$$= \frac{\cancel{8}^1}{2_1} \times -\frac{1}{\cancel{18}_2} = -\frac{1}{4}$$

$$18. -5 \div \left(-3\frac{1}{3}\right) =$$

$$= -\frac{\cancel{5}^1}{1} \div -\frac{\cancel{10}^1}{3} = -\frac{\cancel{5}^1}{1} \times -\frac{3}{\cancel{10}_2} = \frac{3}{2} = 1\frac{1}{2}$$

$$20. -\frac{2}{3} \times \left(-\frac{6}{14}\right) \times \frac{21}{24} \times \left(-\frac{3}{4}\right) \times \frac{8}{9} =$$

$$= -\frac{\cancel{2}^1}{3_1} \times -\frac{\cancel{6}^1}{\cancel{14}_2} \times \frac{\cancel{21}^3}{\cancel{24}_2} \times \left(-\frac{\cancel{3}^1}{4_1}\right) \times \frac{\cancel{8}^2}{\cancel{9}_3} =$$

$$= -\frac{1}{6}$$

Order of Operations with Fractions

Examples

Fraction bar:

$$\frac{-5 + 3^2}{16 - 7(2)} =$$

Step 1: Simplify the numerator.

$$-5 + 3^2 = -5 + 9 = 4$$

Step 2: Simplify the denominator.

$$16 - 7(2) = 16 - 14 = 2$$

$$\frac{-5 + 3^2}{16 - 7(2)} = \frac{4}{2} = 2$$

Step 3: Simplify the fraction.

Fractions:

$$-\frac{3}{4} + \left(-\frac{1}{2}\right)^2 \div \frac{2}{3} =$$

Step 1: Do the exponents first.

$$-\frac{3}{4} + \frac{1}{4} \div \frac{2}{3} =$$

Step 2: Divide (remember that with fractions, you multiply by the reciprocal)

$$-\frac{3}{4} + \frac{1}{4} \times \frac{3}{2} =$$

$$-\frac{3}{4} + \frac{3}{8} =$$

Step 3: Find the lowest common denominator.

$$-\frac{6}{8} + \frac{3}{8} = -\frac{3}{8}$$

Step 4: Add or subtract and simplify if necessary.

Student practice:

1.
$$\frac{-9 + (-6^2) - (-1)}{-3 + (-10) - 1} =$$

Instructor led



2.
$$-\frac{7}{10} + \frac{4}{11} \left(-\frac{1}{4} + \frac{4}{5}\right) =$$



Exercise 5.3

Solve the following numerical expressions.

$$1. \frac{-6 + 3^2 - (-7)}{7 - 9 - 3} = \frac{-6 + 9 + 7}{7 - 9 - 3} = \frac{16 - 6}{-5} = \frac{10}{-5} = -2$$

$$2. \frac{-9 + 18 \div (-3)(-6)}{32 - 4(12) \div 3(2) + 3} = \frac{-9 + -6 \times -6}{32 - 48 \div 3 \times 2 + 3} = \frac{-9 + 36}{32 - 16 \times 2 + 3}$$

$$= \frac{27}{32 - 32 + 3} = \frac{27}{3} = 9$$

$$3. \frac{3}{5} \left(\frac{1}{3} \right) + \frac{2}{5} \left(\frac{3}{4} \right) = \frac{\overset{1}{\cancel{3}}}{5} \times \frac{1}{\cancel{3}_1} + \frac{\overset{2}{\cancel{2}}}{5} \times \frac{\overset{3}{\cancel{4}}_2}{\cancel{4}_2}$$

$$= \frac{1 \times 2}{5 \times 2} + \frac{3}{10}$$

$$= \frac{2}{10} + \frac{3}{10} = \frac{5}{10} = \frac{1}{2}$$

$$4. \left(\frac{7}{8} - \frac{3}{4} \right) \div \frac{3}{2} = \left(\frac{7}{8} - \frac{6}{8} \right) \times \frac{2}{3} = \frac{1}{8}_2 \times \frac{\overset{2}{\cancel{2}}_1}{3} = \frac{1}{6}$$

$$5. \frac{3}{8} \left(\frac{1}{4} + \frac{1}{2} \right) \times \frac{32}{3} = \frac{3}{8} \times \left(\frac{1}{4} + \frac{2}{4} \right) \times \frac{\overset{32}{\cancel{32}}_3}{3}$$

$$= \frac{3}{8}_1 \times \frac{\overset{3}{\cancel{3}}_1}{4}_1 \times \frac{\overset{32}{\cancel{32}}_4}{8}_1 = \frac{3}{1} = 3$$

$$\begin{aligned}
 6. \quad \left(\frac{1}{3} + \frac{1}{6}\right)^2 - 3\left(\frac{2}{3} - \frac{1}{9}\right) &= \left(\frac{2}{6} + \frac{1}{6}\right)^2 - 3\left(\frac{6}{9} - \frac{1}{9}\right) \\
 &= \left(\frac{3}{6}\right)^2 - 3\left(\frac{5}{9}\right) \\
 &= \left(\frac{1}{2}\right)^2 - \frac{2^1}{1} \times \frac{5}{9_3} = \frac{1}{4} - \frac{5}{3} = \frac{2}{12} - \frac{20}{12} \\
 &= -\frac{17}{12} = -1\frac{5}{12}
 \end{aligned}$$

$$\begin{aligned}
 7. \quad 9\left(\frac{1}{3}\right)^3 \times \left(\frac{4}{3}\right)^2 &= 9\left(\frac{1}{27}\right) \times \frac{16}{9} = \frac{9^1}{1} \times \frac{1}{27} \times \frac{16}{9_1} \\
 &= \frac{16}{27}
 \end{aligned}$$

$$\begin{aligned}
 8. \quad \left(\frac{5}{6} - \frac{7}{12}\right) - \frac{3}{4}\left(\frac{1}{3}\right)^2 &= \left(\frac{10}{12} - \frac{7}{12}\right) - \frac{3^1}{4} \times \frac{1}{9_3} \\
 &= \frac{3}{12} - \frac{1}{12} = \frac{2}{12} = \frac{1}{6}
 \end{aligned}$$

$$\begin{aligned}
 9. \quad \left(\frac{2}{3}\right)^2 - \left(\frac{5}{8} - \frac{1}{2}\right) \div \frac{3}{2} &= \frac{4}{9} - \left(\frac{5}{8} - \frac{4}{8}\right) \times \frac{2}{3} \\
 &= \frac{4}{9} - \frac{1}{8_4} \times \frac{2^1}{3} \\
 &= \frac{4 \times 4}{9 \times 4} - \frac{1 \times 3}{12 \times 3} \\
 &= \frac{16}{36} - \frac{3}{36} = \frac{13}{36}
 \end{aligned}$$

Post-Module Assessment and Glossary

Post-Module Assessment

Now that you have completed this module, reassess what you can do against this checklist:

In this module, I will learn how to ...	I can't do this	I can do this with help	I can do this!
1. Solve numerical expressions with negative and positive numbers			
2. Solve numerical expressions using the order of operations			

Glossary for this Module

Arithmetic operations	Addition, subtraction, multiplication, division, etc.
Base	The number the exponent is attached to; in the power 4^2 , the base is 4
BEDMAS	An acronym used to help students remember in what order to perform operations: B rackets, E xponents, D ivision and M ultiplication, A ddition and S ubtraction
Exponent	The number attached to the base that tells you how many times the base is multiplied with itself; in the power 4^2 , the exponent is 2
Integer	A number that is written without a fractional component; it can be positive or negative
Like signs	Signs that are the same
Negative whole numbers	Numbers less than zero, such as -1 , -2 , -3 ...
Numerical expression	A combination of numbers and one or more arithmetic operations; for example, $(4 + 5) \times 8 =$
Order of operations	The rules we follow when calculating numerical expressions

Positive whole numbers	Numbers greater than zero, such as 1, 2, 3, ...
Power	The base and exponent together; for example, 4^2 is a power and means 4×4
Product	The answer when two or more numbers are multiplied together
Quotient	The answer when one number is divided by another
Square root	The opposite of squaring a number. The number in the square root sign, or <i>radical</i> , is a product of a number multiplied with itself; for example $\sqrt{16} = \sqrt{4 \times 4} = 4$ 4 is the result of $\sqrt{16}$, which means that 4 is the square root of 16
Subtracting integers	Add the opposite of the second sign
Unlike signs	Signs that are opposite

Appendix: Exercise Answer Key

Unit 1: Introduction to Signed Numbers

Exercise 1.1

1. $-\$500$ 2. $300, +300$ 3. -7 4. $3, +3$ 5. -20%
6. -7 7. $\$50, +\50 8. -30 9. $7000, +7000$ 10. -12

Exercise 1.2

1. a. -4 should be circled b. 3 should be circled
c. 5 should be circled d. 0 should be circled
e. -1 should be circled
2. a. $<$ b. $>$ c. $<$ d. $<$ e. $<$
f. $>$ g. $<$ h. $>$ i. $<$ j. $>$
3. a. 8 b. 0 c. 15 d. 37 e. 218
f. 45

Unit 2: Addition and Subtraction of Integers

Exercise 2.1

1. 8 2. 17 3. -14 4. -5 5. -14
6. -19 7. 100 8. 10 9. -122 10. -403
11. -10 12. 13 13. -11 14. 5 15. -41
16. -7 17. -19 18. 6 19. 15 20. -39

Exercise 2.2

1. -3 2. -2 3. -17 4. -4 5. -3 6. -3
7. -14 8. 32 9. 0 10. 58 11. -43 12. -42
13. -37 14. 23 15. -49 16. 21

Exercise 2.3

1. The change in temperature was 8°C .
2. No, they did not gain enough yards to keep the ball.
3. Jim has \$28 left in his bank account.
4. The temperature the next day was 4°C .
5. The temperature at 7 p.m. was -25 degrees C.

Unit 3: Multiplication and Division of Signed Numbers**Exercise 3.1**

- | | | | | |
|---------|----------|----------|----------|----------|
| 1. -10 | 2. -48 | 3. -16 | 4. -14 | 5. 7 |
| 6. -18 | 7. -63 | 8. -48 | 9. -49 | 10. 56 |
| 11. -24 | 12. -560 | 13. 64 | 14. -54 | 15. -210 |
| 16. 400 | 17. 56 | 18. -240 | 19. -320 | 20. 540 |

Exercise 3.2

- | | | | | | |
|-------|-------|---------|--------|--------|--------|
| 1. 64 | 2. 16 | 3. -125 | 4. 49 | 5. -32 | 6. -27 |
| 7. 81 | 8. 6 | 9. 8 | 10. 12 | 11. 3 | 12. 5 |

Exercise 3.3

- | | | | | | |
|-------|--------|-------|---------|-------|--------|
| 1. -1 | 2. 4 | 3. 6 | 4. -3 | 5. -6 | 6. 3 |
| 7. -7 | 8. 4 | 9. 3 | 10. -7 | 11. 5 | 12. -9 |
| 13. 5 | 14. -3 | 15. 8 | 16. 200 | | |

Exercise 3.4

- | | | | | | | |
|-------|-------|--------|-------|---------|--------|--------|
| 1. 11 | 2. 10 | 3. 4 | 4. 55 | 5. 24 | 6. 8 | 7. 75 |
| 8. 60 | 9. 15 | 10. 41 | 11. 5 | 12. 100 | 13. 32 | 14. 40 |

Exercise 3.5

1. -20 2. 16 3. 126 4. 14 5. -29 6. 7

Exercise 3.6

1. The salesman travelled $2\,205$ km during the week.
2. Mary will raise $\$108$.
3. Their account balance was $\$1\,197$.
4. The cost for each student would be $\$2\,000$.
5. The temperature was 13°C on June 28.

Unit 4: Operations with Signed Decimals**Exercise 4.1**

1. 23.013 2. -45.75 3. -6.39 4. -6.99 5. -4.279 6. 4.9965
7. -10.63 8. -5.8 9. -237.571 10. -59.23 11. -2.29 12. 1.815
13. -527.7 14. 23.042 15. -24.9 16. 11.7

Exercise 4.2

1. 0.0432 2. -1.84 3. 0.35 4. -0.18 5. -0.798 6. 0.48
7. -8.16 8. 9.0896 9. -3.9 10. 0.47 11. 40.02 12. -36
13. -0.06 14. 516.667 15. -18.889 16. -26.756 17. -0.273 18. 0.1

Exercise 4.3

1. -808.9 2. 2.52 3. 2.12 4. There is $-\$16.08$ in his bank account.

Unit 5: Operations with Signed Fractions

Exercise 5.1

1. $-\frac{1}{2}$
2. $\frac{4}{9}$
3. $\frac{7}{8}$
4. $-\frac{13}{24}$
5. $-\frac{2}{3}$
6. $-1\frac{2}{5}$
7. $-1\frac{11}{18}$
8. $-\frac{5}{18}$
9. $-\frac{2}{5}$
10. $-\frac{1}{8}$
11. $-6\frac{3}{10}$
12. $-2\frac{11}{12}$
13. $-44\frac{2}{21}$
14. $-2\frac{37}{56}$
15. $-10\frac{2}{3}$
16. $-14\frac{1}{2}$
17. $-32\frac{1}{2}$
18. $7\frac{23}{24}$
19. $-\frac{2}{3}$
20. $-1\frac{7}{16}$

Exercise 5.2

1. $\frac{9}{10}$
2. $-\frac{4}{7}$
3. $-\frac{1}{6}$
4. $-\frac{1}{2}$
5. $\frac{4}{5}$
6. $\frac{7}{16}$
7. -6
8. $5\frac{2}{5}$
9. -16
10. -36
11. $-1\frac{1}{2}$
12. $3\frac{1}{3}$
13. $-1\frac{1}{2}$
14. $-\frac{1}{20}$
15. $\frac{1}{12}$
16. $-\frac{1}{4}$
17. $1\frac{1}{2}$
18. $1\frac{1}{2}$
19. $\frac{6}{11}$
20. $-\frac{1}{6}$

Exercise 5.3

1. -2
2. 9
3. $\frac{1}{2}$
4. $\frac{1}{12}$
5. 3
6. $-1\frac{5}{12}$
7. $\frac{16}{27}$
8. $\frac{1}{6}$
9. $\frac{13}{36}$