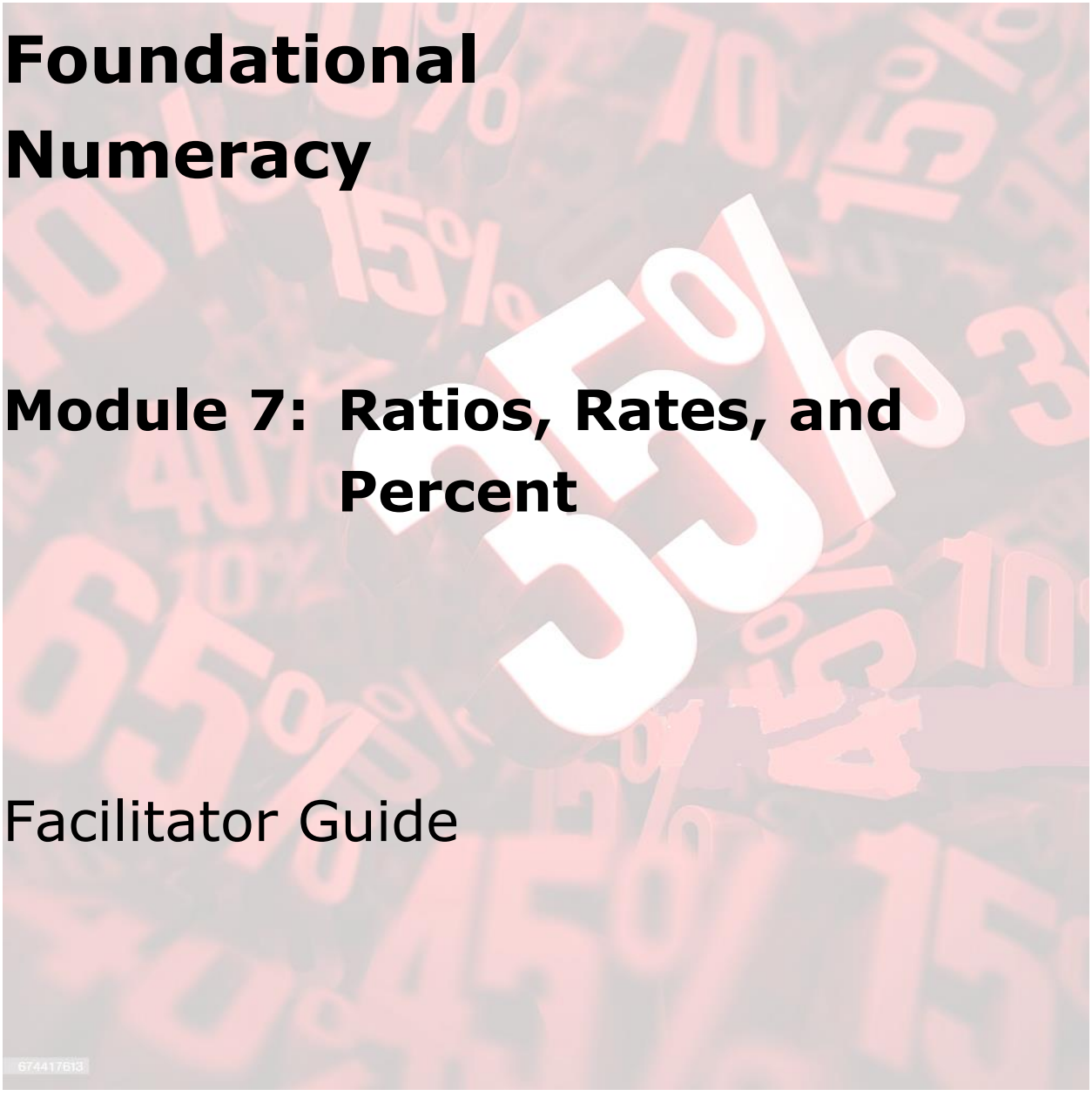
Foundational Numeracy



Module 7: Ratios, Rates, and Percent

Facilitator Guide

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Introduction to the Module

In this module, you will learn to solve problems using proportions and percent in ways that can be applied to everyday problems. In the process, you will develop an understanding of ratios and rates, proportions, and percent. Enjoy your studies!

Important

When you see an object like the one below, you can use a QR code scanner on your phone or tablet, and it will play a video of the math example.

Want to watch a video of this lesson?
Scan the QR Code to the left,
or use the link below:
https://youtu.be/RQ2nYUBVvqI

Note: The facilitator guide mirrors the Learner Guide with a couple of key differences.

- Facilitator notes throughout the module in boxes like this. Include teaching strategies and common errors
- Student Practice doesn't have this bubble. 

The instructor can teach the concept or the learner can watch the video

Specific Learning Outcomes

The table below displays the skills and knowledge you will explore in this module. This is your opportunity to evaluate your own skills to see if you can do these things. At the end of this module, you will be invited to re-evaluate your skills to measure the progress you have made.

In this module, I will learn how to …	I can't do this	I can do this with help	I can do this!
1. Work with ratios and rates			
2. Solve proportions			
3. Work with percent			
4. Change between fractions, decimals, and percent			
5. Solve application problems with proportions and percent			

Essential Skills

The following essential skills are used in this module:

Reading: Understanding materials written in sentences or paragraphs

Numeracy: Using and understanding numbers

Computer use/digital technology: Using computers and other tools such as calculators and phones

Vocabulary: Gaining related vocabulary

Thinking: Finding information, problem solving, decision making, using your memory, planning job tasks and being organized, critical thinking

Module 4: Keywords

Ratio	A comparison of two quantities by division
Rate	The comparison by division of two quantities that are measured in different units
Equivalent	Equal
Proportion	Two or more equivalent ratios
Percent	A ratio that is always given out of 100
Solving proportions	Solving for an unknown value

Unit 1: Ratios and Rates

Ratios

- Ratios compare two numbers and are often in fraction form.
- Order for ratios is very important. Example what is the ratio of ford cars to the total vehicles in a parking lot. Fords need to come first and total vehicles second
- Can be written in 3 ways
- Ford : vehicles
- Ford to vehicles
- $\frac{\text{ford}}{\text{vehicles}}$ **most common form**
- Student will see video link here only **https://youtu.be/RQ2nYUBVvqI**

Think about …

Imagine that a class contains 27 students. There are 12 male students and 15 female students.

With this information, you can make several numerical comparisons about the students in the class. Such a comparison is called a **ratio**. The order of the numbers is very important.

A **ratio** is a **comparison** of one number with another.

- ✦ It is used to show the relationship between the number of one thing and the number of another thing.
- ✦ When you write a ratio, you must keep in mind which number belongs to which thing.
- ✦ The order of the numbers in a ratio is important.

Example 1

How many males are there compared to females? There are 12 males compared to 15 females. **This ratio can be written in the three ways.**

- ✦ This ratio is written as 12 to 15, or 12:15, or $\frac{12}{15}$.

Since males come first in the question, males come first in the ratio.

It is important to reduce ratios to lowest terms.

- ✦ $\frac{12}{15} = \frac{4}{5}$ Therefore, for every 4 males in the class, there are 5 females.

You could also think about this ratio in another way: "How many females are there compared to males?"

- There are 15 females compared to 12 males, so the ratio is 15 to 12, or 15:12, or $\frac{15}{12}$.
- Next, reduce the ratio to lowest terms: $\frac{15}{12} = \frac{5}{4}$

This ratio means that there are 5 females in the class for every 4 males. In this case, the fraction is improper.

Note: When working with ratios, we never convert improper fractions to mixed numbers or whole numbers as the ratio would lose its meaning.

We could also use the information above to compare the number of males to the total number of students in the class or the number of females to the total number of students.

- The comparison of the number of males to all the students in the class is written numerically as 12:27 or $\frac{12}{27}$. This would be read as "12 to 27" or "12 out of 27."
- Next, reduce the ratio to lowest terms: $\frac{12}{27} = \frac{4}{9}$

This ratio means that 4 out of every 9 students are males.

Remember: When ratios are written as fractions, they are usually reduced to their lowest terms, even improper fractions, but you should not change an improper fraction to a whole number or a mixed number.

Example 2

Write a ratio comparing 2 minutes to 60 seconds.

In this example, it is important to convert so that both times use the same unit of measurement.

- 60 seconds is equal to 1 minute.

  Or

- 2 minutes is equal to 120 seconds.

Both conversions are correct.

1. If you use seconds as the unit of measurement:

   120 to 60 = $\frac{120}{60} \div \frac{60}{60} = \frac{2}{1}$ 2 minutes to 60 seconds is a ratio of 2 to 1.

2. If you use minutes as the unit of measurement: 2 to 1 = $\frac{2}{1}$

Student practice: Instructor led

1. There are 10 horses and 5 dogs.
   a. Write a ratio comparing horses to dogs.

   b. Write a ratio comparing dogs to horses.

Scan the QR Code to the left, or use the link below:
https://youtu.be/HpdMJaKaXXc

2. Compare 5 dimes to 4 nickels.

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https://youtu.be/OvD5dXmDBc4

Exercise 1.1

1. Write the following comparisons as ratios reduced to the lowest terms.
   a. A recipe calls for 3 cups of flour and 1 cup of milk. Write the ratio of flour to milk.

   b. There are 16 green marbles and 4 blue marbles in a bag. Write the ratio of blue marbles to the total number of marbles.

   c. The police checked 75 vehicles. They found that 15 vehicles did not meet the safety standards, but 60 of them did. Write a ratio comparing the unsafe vehicles to the safe vehicles.

d. The police checked 120 vehicles. They found that 30 vehicles did not meet the safety standards, but 90 of them did. Write a ratio comparing the unsafe vehicles to the total number of vehicles.

e. At a barbeque, 36 hotdogs and 18 hamburgers were eaten. Write the ratio of hotdogs eaten to hamburgers eaten.

2. Write each of the following as ratios in two ways. Reduce the ratios to lowest terms.

a. The value of 16 dimes to the value of 20 pennies

b. The value of 12 dimes to the value of 36 nickels

c. 15 dogs to 45 cats

d. 0.18 metres to 0.12 metres

e. 20 minutes to 3 hours

Rates

Rates are comparing 2 different units of measure. Example kilometres per hour. Kilometres is one unit of measure and hour is another unit of measure. Both measures must be stated or the rate loses its meaning Km/hr

We normally write the second measure as 1 unit. Example 1 hour, 1 litre, etc

Student will see introductory video here instead https://youtu.be/RQ2nYUBVvqI

Think about …

- Speed in kilometres per hour
- Fuel consumption in kilometres per litre
- Wages in dollars per hour
- Cost in dollars per kilogram

These are all examples of rates. Rates may be expressed as ratios in reduced form. Note that in each example below, the rate is expressed as a simplified ratio with the second term equal to 1.

Examples

Rate	Ratio
1. 50 kilometres (km) per hour (hr)	50 km/1 hr or 50 km:1 hr or 50km/hr
2. 80 kilometres in 2 hours	80 km/2 hr = 40 km:1 hr =40 km/1 hr or 40km/hr
3. 90 cents for 6 grams (g)	90¢/6 g = 15¢/1 g or 15¢:1 g
4. $12 for 3 people	$12/3 people = $4/1 person or $4:1 person

Note: Because the units are different for each part of the rate, they will not cancel and must be kept.

Student practice:

Jayda takes 3 hours to deliver 189 newspapers on her route. What is her rate per hour at which she delivers her papers?

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https://youtu.be/Zm0KaIw-35k

Exercise 1.2

Express each of the following rates as a ratio in simplest form with the second term equal to 1. Be sure to keep your units.

1. 95 cents a dozen

2. 100 kilometres per hour

3. 18 metres for $9

4. 18 kilometres per litre

5. 52 cars per minute

6. 3 000 revolutions in 2 minutes

7. 540 words in 10 minutes

8. 20 tablespoons for each 5 kilograms

9. 80 cents for 10 grams

10. $3.60 for 3 light bulbs

11. 1 000 kilometres in 4 hours

12. 50 homes have 275 clocks

13. 732 beats in 10 minutes

14. 450 kilometres on 25 litres of fuel

15. A ratio of distance travelled to time is called **speed**. If you travel 45 kilometres in 3 hours, what is your rate (speed) in kilometres per hour (km/h)?

Unit 2: Proportions

Proportions (Equivalent Ratios)

- Proportions are 2 ratios that are equal but shown in different terms. Proportions are most commonly written in fraction form
- $\frac{1}{2}=\frac{2}{4}$ is a proportion as they both mean the same but shown in different terms.
- The easiest method to determine if proportions are true is to use cross products.
- In this example 1x4 = 2x2 as both cross products equal 4 the proportion is true.
- If the cross products do not equal then the proportion is false.
- $\frac{1}{2}\neq\frac{2}{5}$ in this case $1x5\neq 2x2$ proportion is false
- **Student will see introductory video her instead** https://youtu.be/USmit5zUGas

A **proportion** is a mathematical statement that **two ratios are equal**.

This is how a proportion is shown: $\frac{15}{10}=\frac{3}{2}$ or 15:10 = 3:2

✦ It is read: "Fifteen is to ten as three is to two."

For example, we know that 6 eggs out of a container of 12 eggs is the same as ½ a dozen eggs.

By equating the ratios, $\frac{6}{12}=\frac{1}{2}$, we form a proportion.

Method 1: Use Equivalent Fractions by Reducing to Lowest Terms

Make **equivalent fractions** to determine whether a proportion is **true**.

If a proportion is true, the fractions are equivalent.

$$\frac{6}{12}=\frac{1}{2}$$

$$\frac{6\div 6}{12\div 6}=\frac{1}{2}$$ These are equivalent fractions.

Example 1

Is the proportion true?

$$\frac{3}{5} = \frac{6}{10}$$

Reduce $\frac{6}{10}$ to lowest terms:

$\frac{6 \div 2}{10 \div 2} = \frac{3}{5}$ The fractions are equivalent.

Thus, $\frac{3}{5}$ and $\frac{6}{10}$ form a true proportion.

Example 2

Is the proportion true?

$$\frac{6}{8} = \frac{18}{21}$$

Reduce each fraction to lowest terms:

$\frac{6}{8} = \frac{3}{4}$ and $\frac{18}{21} = \frac{6}{7}$

$\frac{3}{4} \neq \frac{6}{7}$ Since these two ratios are not equal, we would say the proportion is false.

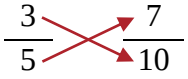
Method 2: Use Cross-Multiplication or Cross Products

Example 1

$2 \times 10 = \mathbf{20}$ $5 \times 4 = \mathbf{20}$

Since the cross products are equal, the proportion is true. So $\frac{2}{5} = \frac{4}{10}$.

Example 2



$3 \times 10 = \mathbf{30}$ $5 \times 7 = \mathbf{35}$

Since the cross products are not equal, the proportion is false. So $\frac{3}{5} \neq \frac{7}{10}$.

Student practice:

Instructor led

1. $\frac{12}{15}$ $\frac{16}{20}$

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https://youtu.be/o5PmulckRYs

2. 7:9 30:45

3. $\frac{1.5}{0.6}$ $\frac{2}{0.8}$

Scan the QR Code to the left, or use the link below:

https://youtu.be/sWqC0vRilvQ

4. $\frac{2\frac{1}{5}}{6}$ $\frac{3\frac{2}{3}}{10}$

Exercise 2.1

Use either method to determine whether each proportion is true or false. If the proportion is true, write = between the ratios. If the proportion is false, write ≠ between the ratios (≠ means *not equal*). Yay

1. $\frac{4}{8}$ $\frac{5}{10}$

2. $\frac{3}{5}$ $\frac{8}{10}$

3. $\frac{3}{7}$ $\frac{17}{42}$

4. $\frac{2}{7}$ $\frac{12}{28}$

5. 6:8 4:3

6. 5:10 15:30

7. $\frac{8}{18}$ $\frac{20}{45}$

8. $\frac{6}{14}$ $\frac{9}{22}$

9. $\frac{18}{16}$ $\frac{2.8}{2.5}$

10. $\frac{4.5}{6}$ $\frac{7.5}{10}$

11. $\frac{0.35}{0.4}$ $\frac{0.7}{0.8}$

12. $\frac{\frac{6}{1}}{\frac{11}{3}}$ $\frac{18}{11}$

13. $\frac{2\frac{5}{8}}{3\frac{1}{4}}$ $\frac{21}{26}$

14. $\frac{3.75}{1\frac{1}{4}}$ $\frac{7.5}{2\frac{1}{2}}$

15. $\frac{2\frac{3}{10}}{8.05}$ $\frac{\frac{1}{4}}{0.9}$

Solving Proportions

The easiest way to find the missing value is often referred to as cross multiply and divide.

This means we multiply the two numbers together that are across from each other and divide by the number that is across from the unknown value. We can check our answer by using cross products.

- $\frac{1}{2}=\frac{x}{6}$ we can show cross multiplication and divide as
- $\frac{1\times 6}{2}=x$ or cross products first $1\times 6=2x$ then divide by the number attached to the unknown $\frac{1\times 6}{2}=\frac{2x}{2}$ then $\frac{1\times 6}{2}=\frac{\cancel{2}x}{\cancel{2}}$ then which means $\frac{6}{2}=x$ so 3 = x or x =3
- Check using cross products $\frac{1}{2}=\frac{3}{6}$ 1x6 =2x3 both products = 6
  - Student will see an intro video here https://youtu.be/USmit5zUGas

Steps for solving a proportion problem using cross-multiplication:

Step 1: Write the first ratio using the information given.

Step 2: Write the proportion using a letter in place of the missing term. Be sure the order of comparison is the same in both the first and second ratios in your proportion.

Step 3: Write the proportion in fraction form. (Try to simplify the ratio before you do all the calculations.)

Step 4: Cross-multiply and set the cross products equal to each other.

Step 5: Divide both sides of the equation by the number with the unknown term.

Step 6: Check by putting your answer back into the original proportion and cross-multiplying.

Solving for a Missing Term in a Proportion

Example 1

For the ratios below to form a true proportion, the missing term must have a specific value.

What is the value of the missing term?

$\frac{4}{5}=\frac{x}{30}$ Write the proportion.

30 ÷ 5 = 6 Raise the fraction $\frac{4}{5}$ to an equivalent fraction with a denominator of 30 by multiplying the numerator and the denominator by the same number.

$\frac{4 \times 6}{5 \times 6} = \frac{24}{30}$ Multiply the numerator and denominator by 6.

Answer: The missing term of the proportion is 24.

Or use cross-multiplication:

$\frac{4}{5} = \frac{x}{30}$ Write the proportion.

$4 \bullet 30 = 5 \bullet x$ Cross-multiply

$120 = 5x$

$\frac{120}{5} = \frac{5x}{5}$ Divide both sides by 5.

$24 = x$

Answer: The value of the missing term is 24.

Check the answer by using cross products.

$\frac{4}{5} = \frac{24}{30}$

$5 \bullet 24 = 4 \bullet 30$

$120 = 120$

The cross products are the same, so the answer ***x* = 24** is correct.

Example 2

Find the value of the missing term.

$\frac{3}{x} = \frac{1}{5}$ Raise the fraction $\frac{1}{5}$ to an equivalent fraction with a numerator that can be divided by 3.

$\frac{1 \times 3}{5 \times 3} = \frac{3}{15}$

Answer: The value for the missing term that will yield a true proportion is **15**.

Or cross-multiply:

$$\frac{3}{x} = \frac{1}{5}$$

$3 \bullet 5 = x \bullet 1$ Cross multiply

$$\frac{15}{1} = \frac{1x}{1}$$ Then divide.

$$15 = x$$

Answer: $x = 15$

Student practice:

Instructor led

1. $\frac{2}{3} = \frac{x}{12}$

SCAN ME

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https://youtu.be/gwfG7RaOxX4

2. $\frac{8}{36} = \frac{10}{x}$

3. $\frac{0.06}{x} = \frac{0.3}{0.4}$

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https://youtu.be/NgCzHJxtlUs

4. $\frac{\frac{1}{4}}{\frac{1}{2}} = \frac{x}{8}$

Exercise 2.2

1. $\frac{1}{5} = \frac{x}{25}$

2. $\frac{16}{x} = \frac{8}{3}$

3. $\frac{3}{2} = \frac{x}{4}$

4. $\frac{8}{x} = \frac{4}{5}$

5. $\frac{12}{x} = \frac{16}{4}$

6. $\frac{9}{x} = \frac{3}{4}$

7. $\frac{28}{x} = \frac{4}{7}$

8. $\frac{21}{75} = \frac{x}{25}$

9. $\frac{15}{6} = \frac{50}{x}$

10. $\frac{17}{10} = \frac{8.5}{x}$

11. $\frac{2.6}{x} = \frac{13}{7.8}$

12. $\frac{0.4}{0.9} = \frac{4.8}{x}$

13. $\frac{x}{\frac{3}{10}} = \frac{\frac{20}{9}}{1}$

14. $\frac{15}{1\frac{2}{3}} = \frac{9}{x}$

15. $\frac{\frac{3}{20}}{0.1} = \frac{0.03}{x}$

Solving Application Problems with Proportions

Using Proportions to Solve Word Problems

Common errors learners make when setting up proportions is not putting the numbers in the correct place in the ratio.

$\frac{2\text{kms}}{1\text{hour}} = \frac{x\text{ hours}}{8\text{kms}}$ this is **incorrect** if you used cross products the answer would be 16hours instead of 4

Correct Method

- Jim can run 2kms in 1 hour. How far will it take him to run 8kms?
  - $\frac{2\text{kms}}{1\text{hour}} = \frac{8\text{kms}}{x\text{ hours}}$ notice kilometers are on top and hours are on the bottom.
- It is important to keep the same units of measure in the same place in the ratios.
- Student will see intro video https://youtu.be/s8_14yxp1lQ

1. Read the problem at least twice.
2. Decide what you have to find (the missing term) and assign a letter to represent it.
3. Set up a proportion using the information given.
4. Solve the proportion.
5. Check using cross products.
6. Write the answer in sentence form, including the appropriate units of measure.

Example 1

At the rate of 5 articles for 20 cents, find the cost of 4 articles.

Solution:

The question asks us to **find the cost**. Use the letter C to represent cost.

$\frac{5\text{ articles}}{20¢} = \frac{4\text{ articles}}{C}$ Set up a proportion.

Note: The top and bottom terms must correspond (represent the same things). In this example, the top terms represent the number of articles, and the bottoms terms represent cost.

$\frac{5}{20} = \frac{4}{C}$ Cross-multiply.

$5 \times C = 4 \times 20$

$5C = 80$

$\frac{5C}{5} = \frac{80}{5}$ Divide both sides by 5 to find the unknown value.

$C = 16$ The value of C is 16. Write the answer in sentence form, including the proper units.

Check $\frac{5}{20} = \frac{4}{16}$ use cross products $5 \times 16 = 4 \times 20$

80 = 80 both products are 80, the answer is correct

Answer: 4 articles cost 16 cents.

Student practice: Instructor led

1. A grocery store sells 3 boxes of cereal for $11.50. At this rate, how much would 9 boxes of cereal cost?

Scan me

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https://youtu.be/UY7jPpfXygw

2. John can drive 150 kilometres in 2 hours. At this rate, how long would it take him to drive 375 kilometres?

Exercise 2.3

Solve each of the following problems. Round the answers to the nearest tenth as necessary.

1. A recipe uses 3 pounds of hamburger to make a casserole large enough to feed 13 people. Using the same recipe, how many people could be fed with 9 pounds of hamburger?

2. Joe can run 100 metres in 12 seconds. If he maintains the same speed, how far could he run in 36 seconds?

3. A large container of soap cost $9 for 12 litres. What could you expect to pay for 4 litres?

4. Gertrude walked 9 blocks in 15 minutes. At the same rate, how long will it take her to walk 18 blocks?

5. A truck travelled 250 kilometres in 5 hours. At this speed, how long would it take to complete a trip of 650 kilometres?

6. If the scale of distance of 1 centimetre on a map represents an actual distance of 10 kilometres, what actual distance does a scale distance of 5.7 centimetres represent?

7. The directions on the lawn fertilizer say to spread 1.7 kilograms over 100 square metres of lawn. How much fertilizer is needed for a lawn measuring 130 square metres?

8. The taxes on a property valued at $300 000 are $5 000. At the same rate of taxation, what would the taxes be on the smaller lot down the street that is valued at $240 000?

9. The class spends 6 hours a week on math and 8 hours on English. At this rate, if the class spent 10½ hours on math, how much time would be spent on English?

10. At 4 p.m., Glen's shadow is 2.15 metres long. His height is 1.85 metres. A tree nearby has a shadow that is 7.2 metres long. How tall is the tree? Round your answer to the nearest hundredth of a metre.

Unit 3: Percent

Percent is always out of 100

Percent can be written as a fraction or a decimal.

Think about…

When we buy something at a store, we always pay more than the posted price. If we buy something that costs $1.00, we pay $0.05 more in tax (GST), so the final cost is $1.05. In Canada, we pay 5% GST on all goods.

Percent is a ratio that has many applications—interest rates, discounts, sales tax, and marks are all expressed as a percent.

- **Percent means "out of a hundred."**
- 75% means $\frac{75}{100}$ or 0.75

Percent as a Decimal

We are really dividing the percent by 100 which is the same as moving the decimal two places to the left and dropping the percent sign. If there is no decimal in a number it is always at the end.

- The Common errors when writing a percent as a decimal is
- learners will drop the percent sign forget to move the decimal
- Or the learner will drop the percent sign and move the decimal the wrong way (2 places to the write rather than 2 places to the left)
- Student will see introductory videos https://youtu.be/3EB75Dy4dFE and https://youtu.be/s_DA3qW9pjc

Percent can be written in decimal form by dividing by 100 and dropping the % sign.

- Dividing by 100 is the same as moving the decimal two places to the left.

Decimals can be written in % form by multiplying by 100 (the same as moving the decimal two places to the right) and adding the percent sign.

- 0.75 = 75%

Example 1

Write 20% as a decimal.

20% = 20 ÷ 100 = **0.2** Two decimal places means hundredths.

Example 2

Write 0.4 as a percent.

0.4 × 100 = 40 Multiply by 100 or move the decimal two places to the right.

= **40%** Drop the decimal point and add the percent sign.

Student practice:

Instructor led

1. Write 45% as a decimal.

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https://youtu.be/up-Bhg8Dv7s

2. Write 0.234 as a percent.

Exercise 3.1

1. Write each of the following as a percent.

   a. 0.56 b. 0.6 c. 0.04

   d. 0.21 e. 0.625 f. 0.855

   g. 1.7 h. 2.35 i. 1.23

2. Write each of the following in decimal form.

   a. 24% b. 33% c. 5%

   d. 16.7% e. 120% f. 200%

   g. 44% h. 125% i. 87.5%

Percent and Fractions

- To write a percent as a fraction put the percent over 100 and drop the % sign.

Example 3% = $\frac{3}{100}$

- Fractions must also be reduced to lowest term
  - If a percent has a decimal in it we must move the decimal to the end of the number and add zeros to the denominator as decimals and fractions don't mix this is really multiplying the numerator and denominator by 10
  - 9.5% = $\frac{9.5}{100} = \frac{9.5 \times 10}{100 \times 10} = \frac{95}{1000}$ now reduce to lowest terms $\frac{95 \div 5}{1000 \div 5} = \frac{19}{200}$
- Common errors are not dropping the percent sign and not reducing fractions to lowest terms.

Example 1

There are two ways to write a fraction as a percent.

Write ¾ as a percent.

1. Change the fraction to a decimal by dividing. Then move the decimal two places to the right by multiplying by 100.

   $\frac{3}{4} = 3 \div 4 = 0.75$

   $0.75 \times 100 =$ **75%**

Or

2. First write the fraction as a proportion. Remember that percent (%) is always out of 100.

   $\frac{3}{4} = \frac{n}{100}$

   $3 \bullet 100 = 4n$ Then cross-multiply and divide.

   $\frac{300}{4} = \frac{4n}{4}$

   $75 = n$

   ¾ = **75%**

Example 2

Write 20% as a fraction.

$\frac{20}{100} = \frac{1}{5}$

To write 20% as a fraction, put 20 over 100 and drop the % sign.
Then reduce the fraction to lowest terms.

Student practice:

Instructor led

Scan the QR Code to the left, or use the link below:
https://youtu.be/aBRqRB_0uhQ

1. Write 18% as a fraction.

2. Write $\frac{3}{5}$ as a percent.

3. Write 35% as a fraction and a decimal.

Exercise 3.2

1. Write each fraction as a percent. Round your answer to one decimal place when necessary.

a. $\frac{7}{10}$ b. $\frac{2}{5}$ c. $\frac{1}{4}$

d. $\frac{1}{2}$ e. $\frac{3}{8}$ f. $\frac{4}{9}$

g. $\frac{7}{8}$ h. $\frac{3}{2}$ i. $\frac{2}{3}$

2. Write each percent as a fraction or mixed number and reduce to lowest terms.

a. 24% b. 33% c. 6%

d. 120% e. 44% f. 10%

g. 16% h. 87.5% 16⅔%

Fractions, decimals and percent all mean the same amount they are just shown in different ways.

- Convert fractions to decimals divide numerator by denominator
  - $\frac{3}{5} = 3 \div 5 = 0.6$
- To convert decimal to fraction write the decimal number over 10, 100, 1000, etc and drop the decimal sign then reduce the fraction to lowest terms
- $0.6 = \frac{6}{10} = \frac{6 \div 2}{10 \div 2} = \frac{3}{5}$
- Notice $\frac{3}{5} = 0.6$

3. **Extra Practice: Fractions, Decimals, and Percent**

Complete the table below by filling in the empty blocks.

	Fraction	Decimal	Percent
a.	$\frac{1}{4}$		
b.		0.5	
c.			75%
d.	$\frac{1}{8}$		
e.		0.375	
f.			62%

Solving Problems with Percent: Part I

Percent Problems

- The below formula always works for solving percent problems

$$\frac{\text{is (part)}}{\text{of (whole)}} = \frac{\%}{100}$$

- Common errors – putting the numbers in the wrong spot normally mixing up the part and whole numbers
- Focus on side percent over 100 never changes
- Leaners will have this video **https://youtu.be/Uo8HgcyfRFI**

You can use the following formula to solve all percent problems:

$$\frac{\text{is (part)}}{\text{of (whole)}} = \frac{\%}{100}$$

Example 1

Find 30% of 40.

Before a percent problem can be solved, the percent must be expressed as a decimal or a fraction, or you can use the above formula.

30% of 40

$= 0.30 \times 40$

$= 12$

or

$$\frac{\text{is (part)}}{\text{of (whole)}} = \frac{\text{percent}}{100}$$

$$\frac{x}{40} = \frac{30}{100}$$

$$100 \bullet x = 30 \bullet 40$$

$$100x = 1200$$

$$\frac{100x}{100} = \frac{1200}{100}$$

$$x = 12$$

Answer: 30% of 40 is 12.

Example 2

65% of a number is 130. What is the number? Let n represent the unknown number.

65% of n is 130

$$0.65n = 130$$

$$\frac{0.65n}{0.65} = \frac{130}{0.65}$$

$$n = 200$$

or

$$\frac{\text{is (part)}}{\text{of (whole)}} = \frac{\text{percent}}{100}$$

$$\frac{130}{n} = \frac{65}{100}$$

$$130 \bullet 100 = 65 \bullet n$$

$$\frac{13\,000}{65} = \frac{65n}{65}$$

$$200 = n$$

Answer: 65% of 200 is 130.

Example 3

What percent of 50 is 75? Let n represent the unknown percent.

n% of 50 is 75

$$0.01n \times 50 = 75 \qquad \leftarrow n\% = n \text{ hundredths or } 0.01n$$

$$0.5n = 75$$

$$\frac{0.5n}{0.5} = \frac{75}{0.5}$$

$$n = 150 \text{ or } 150\%$$

Or cross-multiply and divide:

$$\frac{75}{50} = \frac{\%}{100}$$

$$\frac{75}{50} = \frac{n}{100}$$

$$75 \bullet 100 = 50n$$

$$\frac{7500}{50} = n$$

$$150 = n$$

Answer: 150% of 50 is 75.

Student practice:

Instructor led

1. 20% of what number is 12?

Scan the QR Code to the left, or use the link below:
https://youtu.be/421JZ1h1LQo

2. Tim wrote a test and scored 80%. If the test had 20 questions and each question was worth 1 mark, how many questions did Tim answer correctly?

3. Stores offer discounts to attract customers. One store is offering a 40% discount on all their appliances. If a refrigerator regularly sells for $1 200, what would the sale price be?

Exercise 3.3

1. Find 25% of 40.

2. 60% of what number is 24?

3. What percent of 20 is 16?

4. 150% of what number is 30?

5. Find 10% of $460.

6. 0.5% of what number is 5?

7. What percent of 125 is 25?

8. 75% of 12 is what number?

9. What is 16% of 34?

10. What is 90% of 75?

11. The seller pays a 5% commission for selling a house. What is the commission for the sale of a house valued at $110 000?

12. Calculate the sale price for a camera regularly priced at $150, on sale at 20% off.

13. Josephine paid $150 for a camera that was on sale for 25% off. What was the regular price of the camera?

14. In telethons and other fundraising events, records show that only about 80% of the money pledged is actually collected. Local telethon organizers need to raise $12 000. To raise $12 000, their goal for pledges should be what amount?

Solving Problems with Percent: Part II

Problems that ask you to find the percent of increase or decrease

Common error is the learner may not understand if it is an increase or a decrease.

They also not find the difference first. Not finding the difference from the initial amount and the new amount.

- Example temperature went from 10 degrees to 8 degrees. Sometime learners will put the numbers in $\frac{10}{8}=\frac{\%}{100}$ that is not correct. The proper way to find the difference first 10-8=2 then $\frac{2}{10}=\frac{\%}{100}$ as the price dropped it is a decrease.

Students will have an intro video here **https://youtu.be/GQJiADE9pFw**

Solving this type of problem often involve twos steps:

Step 1: Find the amount of change (either increase or decrease) by finding the difference between the two amounts given. Subtract to find the difference.

Step 2: Find the percentage of increase or decrease. Always compare the change (the amount of increase or decrease) to the amount before the change (the original amount) using this proportion:

$$\frac{\text{amount of change or difference}}{\text{original amount}} = \frac{\%\,\text{increase or decrease}}{100}$$

Example 1

The rent went from $375 a month to $427.50 a month. By what percent did the rent increase?

Step 1: Find the change (the amount of increase) by finding the difference between the amounts.

$427.50 – 375 = $52.50.
The amount of increase is $52.50.

Step 2: Find the percent of increase.

The amount of increase is $52.50.
The original amount (the amount before the increase) is $375.

$$\frac{\text{difference}}{\text{original}} = \frac{\text{percent}}{100}$$

$$\frac{52.50}{375} = \frac{n}{100}$$

$$5\,250 = 375n$$

$$\frac{5\,250}{375} = \frac{375n}{375}$$

$$14 = n$$

Answer: The rent increased by 14%.

Example 2

The hours of operation of the Tim Horton's at the college were reduced from 35 hours a week to 28 hours a week. By what percent did the operating hours decrease?

Step 1: Find the amount change (a decrease) by finding the difference between the amounts.

35 hours – 28 hours = 7 hours
The amount of decrease is 7 hours.

Step 2: Find the percent decrease.

The decrease is 7 hours.
The original amount is 35 hours.

What percent is 7 of 35?

$$\frac{\text{difference}}{\text{original}} = \frac{\text{percent}}{100}$$

$$\frac{7}{35} = \frac{\%}{100}$$

$$\frac{7}{35} = \frac{n}{100}$$

$$7 \bullet 100 = 35n$$

$$20 = n$$

Answer: The operating hours decreased by 20%.

Problems that require solving for price plus GST

Learners may not understand GST, and what it means.

Leaners may forget to add the amount of GST to the price but add 5 instead

- Final price of an object in Alberta is price plus GST.
- **Price + GST = Total Price**
- GST is always 5%

Example 3

Jane bought a pair of pants costing $59.99. At the checkout counter, she is charged the price plus GST (5%). How much did Jane pay for the pants?

Step 1: Calculate 5% of the cost to find the GST.

$$\frac{x}{59.99} = \frac{5}{100}$$

$$100x = 5 \bullet 59.99$$

$$\frac{100x}{100} = \frac{299.95}{100}$$

$$x = 2.9995$$

$$x = 3$$

5% of the cost (the GST) is $3.

Step 2: Add the GST to the initial price to find the total cost.

$59.99 + $3.00 = $62.99

Answer: Jane paid $62.99 for the pants.

Student practice: Instructor led

Scan the QR Code to the left, or use the link below:
https://youtu.be/hxDknrS9jTU

1. The cost of hourly parking went from $9 per day to $12 per day. By what percent did the parking fees increase?

2. Jamal bought an iPhone for $730.00. Find the amount of GST and the total cost of the iPhone.

Exercise 3.4

1. Jennifer's wage recently went from $10.00 an hour to $10.50 an hour. By what percent did her wages increase?

2. The enrolment in the afternoon bowling league went from 40 bowlers to 50 bowlers for the spring session. By what percent did the number of bowlers increase?

3. A regular toilet uses 20 litres of water per flush. A new low-flow toilet uses only 6 litres per flush. What is the percent savings of water per flush if the new toilet is used?

4. Sixteen litres of raw strawberries only produced 13.5 litres of cooked strawberries. By what percent did the amount of strawberries shrink?

5. A car dealership gives a special deal if the customer does not have a trade-in and pays cash. The dealer will only charge $10 650 for a car listed at $12 000. What is the percent savings in this deal?

6. A watch is regularly priced at $90.00. It is on sale for 10% off.

   a. How much is the discount?

   b. What is the sale price?

7. Sylvia receives a weekly salary of $280.00 plus a commission of 8% on all the sales she makes over $2 000.00. Last week, she sold $5 600.00 worth of merchandise. How much money did she make?

8. For each item in the table below, find the GST (5%), then calculate the total cost.

	Item	Cost	GST (5%)	Total Cost
a.	T-shirt	$14.99		
b.	New car	$27 890.00		
c.	60″ flat-screen TV	$699.00		

Post-Module Assessment and Answer Key

Post-Module Assessment

Now that you have completed this module, reassess what you can do against this checklist:

In this module, I will learn how to …	I can't do this	I can do this with help	I can do this!
1. Work with ratios and rates			
2. Solve proportions			
3. Work with percent			
4. Change between fractions, decimals, and percent			
5. Solve application problems with proportions and percent			

Glossary of Terms

Ratio	A comparison of two quantities by division
Rate	The comparison by division of two quantities that are measured in different units
Equivalent	Equal
Proportion	Two or more equivalent ratios
Percent	A ratio that is always given out of 100
Solving proportions	Solving for an unknown value

Answer Key

Exercise 1.1

1. a. $\frac{3}{1}$ or 3:1 b. $\frac{4}{20} = \frac{1}{5}$ or 1:5 c. $\frac{15}{60} = \frac{1}{4}$ or 1:4

   d. $\frac{30}{120} = \frac{1}{4}$ or 1:4 e. $\frac{36}{18} = \frac{2}{1}$ or 2:1

2. a. $\frac{160}{20} = \frac{8}{1}$ or 8:1 b. $\frac{120}{180} = \frac{2}{3}$ or 2:3 c. $\frac{15}{45} = \frac{1}{3}$ or 1:3

   d. $\frac{0.18}{0.12} = \frac{3}{2}$ or 3:2 e. $\frac{20}{180} = \frac{1}{9}$ or 1:9

Exercise 1.2

1. 95¢:1 dozen
2. 100 km:1 h
3. 18:9 = 2 m:$1
4. 18 km:1 L
5. 52 cars:1 min
6. 1 500 revolutions:1 min
7. 540:10 = 54 words:1 min
8. 20:5 = 4 tbsp:1 kg
9. 80:10 = 8¢:1 g
10. $3.60:3 = $1.20:1 light bulb
11. 1 000:4 = 250 km:1 h
12. 274:50 = 5.5 clocks:1 home
13. 732:10 = 73.2 beats:1 min
14. 450:25 = 18 km:1 L
15. 45:3 = 15 km/h

Exercise 2.1

1. =	2. ≠	3. ≠	4. ≠	5. ≠
6. =	7. =	8. ≠	9. ≠	10. =
11. =	12. =	13. =	14. =	15. ≠

Exercise 2.2

1. $x = 5$
2. $x = 6$
3. $x = 6$
4. $x = 10$
5. $x = 3$
6. $x = 12$
7. $x = 49$
8. $x = 7$
9. $x = 20$
10. $x = 5$
11. $x = 1.56$
12. $x = 10.8$
13. $x = \frac{2}{3}$
14. $x = 1$
15. $x = 0.02$ or $\frac{1}{50}$

Exercise 2.3

1. You could feed 39 people.
2. Joe could run 300 metres.
3. You could expect to pay $3.
4. It will take her 30 minutes.
5. It would take 13 hours.
6. It represents 57 kilometres.
7. You would need 2.21 kilograms of fertilizer.
8. The taxes would be $4000.
9. 14 hours would be spent on English.
10. The tree is 6.2 metres tall.

Exercise 3.1

1. a. 56% b. 60% c. 4% d. 21% e. 62.5%
   f. 85.5% g. 170% h. 235% i. 123%
2. a. 0.24 b. 0.33 c. 0.05 d. 0.167 e. 1.2
   f. 2 g. 0.44 h. 1.25 i. 0.875

Exercise 3.2

1. a. 70% b. 40% c. 25% d. 50% e. 37.5%
   f. $44\frac{4}{9}\%$ or 44.4% g. 71.4% h. 150% i. $66\frac{2}{3}\%$ or 66.7%
2. a. $\frac{6}{25}$ b. $\frac{33}{100}$ c. $\frac{3}{50}$ d. $\frac{6}{5}$ or $1\frac{1}{5}$ e. $\frac{11}{25}$
   f. $\frac{1}{10}$ g. $\frac{4}{25}$ h. $\frac{7}{8}$ i. $\frac{1}{6}$

3.

	Fraction	Decimal	Percent
a.	$\frac{1}{4}$	0.25	25%
b.	$\frac{1}{2}$	0.5	50%
c.	$\frac{3}{4}$	0.75	75%
d.	$\frac{1}{8}$	0.125	12.5%
e.	$\frac{3}{8}$	0.375	37.5%
f.	$\frac{31}{50}$	0.62	62%

Exercise 3.3

1. 10
2. 40
3. 80%
4. 20
5. $46
6. 1000
7. 20%
8. 9
9. 5.4 or 5.44
10. 67.5
11. $5 500
12. $120
13. $200
14. $15 000

Exercise 3.4

1. Jennifer's wages increase by 5%.
2. The number of bowlers increased by 25%.
3. There is a 70% savings of water per flush.
4. The amount of strawberries shrunk by 15.625%.
5. The customer would save 11.25%.
6. a. The discount is $9.
   b. The sale price is $81.
7. Sylvia made $568.
8. a. GST = $0.75
   Total Cost = $15.74
   b. GST = $1 394.50
   Total Cost = $29 284.50
   c. GST = $34.95
   Total Cost = $733.95