NORQUEST COLLEGE

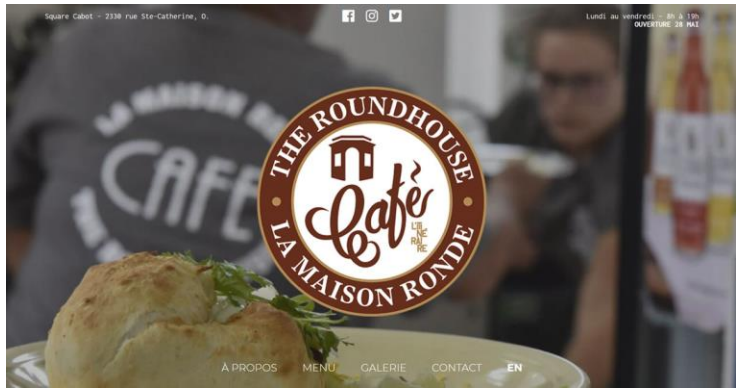
Unit 3

Part 1: Ratios

Ratio

A **ratio** says how much of one thing there is compared to another thing

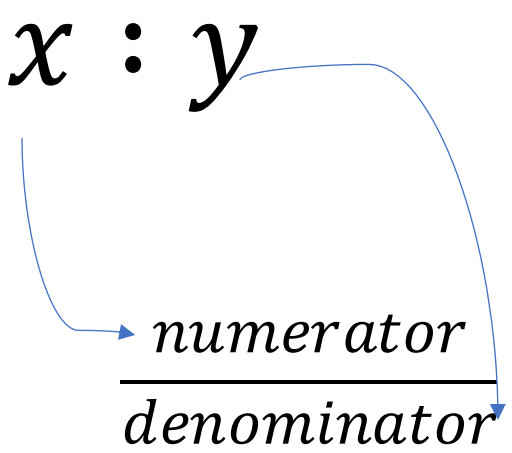
The People of Foundational Literacy: Winston Swappie



Winston's recipe for pancakes at his café in Montreal calls for 2 cups of flour for ever 1 cup of water

Fractional Notation	To Notation	**Colon Notation**
$\frac{2}{1}$	2 to 1	$2:1$
All three ratios above are read as, "the ratio of 2 to 1"		

Writing Ratios as Fractions



Ratios as Fractions

$8 : 5$ $2 : 7$ $5 : 25$

$\frac{8}{5}$ $\frac{2}{7}$ $\frac{5}{25} \begin{matrix} \div 5 \\ \div 5 \end{matrix} = \frac{1}{5}$

Always put your answers in **LOWEST TERMS / SIMPLIFIED FORM**

Writing Ratios in Simplest Form

Write the ratio of $15 to $10 as a fraction in simplest form.

$$15 : 10$$

$$\frac{15}{10} \begin{matrix} \div 5 \\ \div 5 \end{matrix} = \frac{3}{2} \qquad 3 : 2$$

Writing Ratios in Simplest Form

Write the ratio in simplest form.

$$2\frac{2}{3} : 5\frac{3}{5}$$

$$\frac{2\frac{2}{3}}{5\frac{3}{5}}$$

$$2\frac{2}{3} \div 5\frac{3}{5}$$

$$\frac{8}{3} \div \frac{28}{5}$$

$$\frac{8}{3} \times \frac{5}{28}$$

$$\frac{2 \cdot 2 \cdot 2}{3} \times \frac{5}{2 \cdot 2 \cdot 7}$$

$$\frac{\cancel{2} \cdot \cancel{2} \cdot 2 \cdot 5}{3 \cdot \cancel{2} \cdot \cancel{2} \cdot 7}$$

$$\frac{2 \cdot 5}{3 \cdot 7}$$

$$= \frac{10}{21} \qquad 10 : 21$$

Rate

A **rate** is a special kind of ratio that is commonly shown in fractional form

These rates are used to compare different kinds of quantities

For a typical minimum wage employee in Alberta, they would make $120 in an 8 hour shift. This rate is shown in fractional form:

$$\frac{\$120}{8\ hrs} = \frac{\$15}{1\ hr}$$

A person covered a distance of 440 km over the course of 4 hours

$$\frac{440\ km}{4\ hrs} = \frac{110\ km}{1\ hr}$$

Unit Rate

where the denominator is 1

How: divide the “top” and “bottom” by the denominator

Unit Rate

A store charges $220 for 1 oz. (or 28 grams) of wheat seed. What is the unit price in dollars per gram? Round the price to the nearest cent.

$$\frac{\$220}{28\ \text{g}} \overset{\div 28}{\underset{\div 28}{=}} \frac{?}{1\ g}$$

$$220 \div 28 = 7.85714 \qquad = \$\mathbf{7.86}$$

Proportions

A **proportion** is used to show that two ratios or rates are equal. Creating equivalent proportions and determining equivalency is the exact same process as determining equivalency of fractions

Ratio # 1	Ratio #2
$3 : 4$	$75 : 100$

"3 is to 4 as 75 is to 100"

$$\frac{3}{4} = \frac{75}{100}$$

Cross-Products

$$3 \times 100 = \mathbf{300}$$

$$4 \times 75 = \mathbf{300}$$

Same products: Equivalent **OR** **Different products: Not Equivalent**

True or False

$4\frac{1}{5}$ is to $2\frac{1}{3}$ as $3\frac{3}{10}$ is to $1\frac{5}{6}$

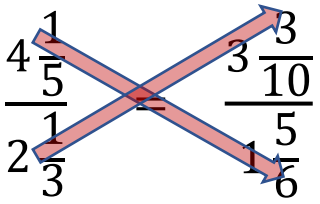
$4\frac{1}{5} : 2\frac{1}{3}$ as $3\frac{3}{10} : 1\frac{5}{6}$

Cross Product #1

$4\frac{1}{5} \times 1\frac{5}{6}$

$\frac{\cancel{21}^{7}}{5} \times \frac{11}{\cancel{6}_{2}}$

$\frac{77}{10}$ *or* 7.7



Cross Product #2

$2\frac{1}{3} \times 3\frac{3}{10}$

$\frac{7}{\cancel{3}_{1}} \times \frac{\cancel{33}^{11}}{10}$

$\frac{77}{10}$ *or* 7.7

Same products: Equivalent

Solving for Unknowns – Still involves Cross Multiplying!

Solve for x

$$\frac{x}{4} = \frac{16}{32}$$

$$32(x) = 4(16)$$

$$\frac{32x}{32} = \frac{64}{32}$$

$$\boldsymbol{x = 2}$$

$$\frac{8}{x} = \frac{5}{9}$$

$$8 \cdot 9 = x(5)$$

$$\frac{72}{5} = \frac{5x}{5}$$

$$\boldsymbol{x = \frac{72}{5} \quad or \quad 14\frac{2}{5}}$$

Discuss: How would you **verify** your answer?

You would find the Cross Products

Same products: Equivalent OR Different products: Not Equivalent

Solving for Unknowns

$$\frac{x}{4\frac{1}{3}} = \frac{4\frac{1}{2}}{1\frac{3}{4}}$$

$$\frac{x}{\frac{13}{3}} = \frac{\frac{9}{2}}{\frac{7}{4}}$$

$$x \cdot \frac{7}{4} = \frac{9}{2} \cdot \frac{13}{3}$$

$$\frac{7}{4}x = \frac{117}{6}$$

$$\frac{\frac{7}{4}x = \frac{117}{6}}{\frac{7}{4} \quad \frac{7}{4}}$$

$$x = \frac{117}{6} \div \frac{7}{4}$$

$$x = \frac{\overset{39}{\cancel{117}}}{\underset{1}{\cancel{6}}} \times \frac{\cancel{4}^{\,2}}{7}$$

$$x = \frac{39}{1} \times \frac{2}{7}$$

$$\boldsymbol{x = \frac{78}{7} \quad or \quad 11\frac{1}{7}}$$

Word Problems

The standard dose of an antibiotic is 4 cc (cubic centimeters) for every 25 lbs (pounds) of body weight. At this rate, find the standard dose for a 140 lb woman.

Rate: 4 cc for every 25 lbs

Ratio $= 4 \text{ cc} : 25 \text{ lbs}$ or $\frac{4 \text{ cc}}{25 \text{ lbs}}$

Woman in Question: 140 lbs

What the 140lb woman wants = how many CC's (x) for a 140lb woman

By knowing the original rate and weight of the woman, we are able to create a proportion where we solve for one of the variables. Note: You must pay attention to where you place your numbers in the proportion as the units of the proportion need to correspond at the same level (at either the denominator or numerator)

$$\frac{4 \text{ cc}}{25 \text{ lbs}} = \frac{x}{140 \text{ lbs}}$$

$$\frac{25x}{25} = \frac{560}{25}$$

$$x = 22.4$$

Consistency is key: In this solution, the cc's are kept in the numerator, and the lbs are kept in the denominator.

If you do not stay consistent with you units, you will create incorrect proportions (disproportions) that derail your progress in the question.

∴ THE STANDARD DOSE FOR A 140LB WOMAN IS 22.4 CC

Word Problems

The People of Foundational Literacy: McKayla McMurray Prescott

McKayla has recently gained an interest in extreme couponing. Not only does it save a lot of money, but it gives her something to do as she's become very bored during the pandemic.

Today, she has a big haul to do at Wal-Mart and she is pondering the following scenario…

"I need to buy some healthy breakfast cereals for my children. Today, I have a coupon for 69 cents off Froot Loops and a coupon for 35 cents off Cap'n Crunch. The Froot Loops cost $2.65 for 11.5 oz. A box of Cap'n Crunch costs $3.67 for 17.5 oz. Assuming I use a coupon, **which cereal is the cheaper buy?"**

Unit 3

Part 2:%

Percent

Percent: per one hundred; uses the symbol "%"

$$\frac{\text{PER}}{100} \quad \text{is the same thing as} \quad \frac{\qquad}{100}$$

Percent is just a way of showing a fraction that has a denominator of…

This also means that percent represents a ratio of $x : 100$ (something TO 100)

$$29\% \qquad \frac{29}{100} \qquad 0.29 \qquad 29 : 100$$

Percent in the World

- Kobe Bryant of the Los Angeles Lakers had a shooting percentage of **35.8%** in his final season in the NBA
- **10%** of people are left handed
- Some cats spend **66%** of their life asleep
- **25%** of all the bones in your body are contained in your feet

- Back on 9/21/2021, findings from the Survey on COVID-19 and Mental Health (SCMH) indicated that one in four (**25%**) Canadians aged 18 and older screened positive for symptoms of depression, anxiety or posttraumatic stress disorder (PTSD) in spring 2021, up from one in five (**20%**) in fall 2020 (Statistics Canada, 2021)

https://www150.statcan.gc.ca/n1/daily-quotidien/210927/dq210927a-eng.htm

Decimals and Fractions → %

Converting Decimal to %

To write a decimal as a percent, multiply the decimal by 100 then attach a "%" to that product. *(Note: to multiply a decimal by 100, just move the decimal point 2 places to the RIGHT of its original spot*)

Write 0.03 as a percent

0.03×100 **3%**

Write 0.9 as a percent

0.9×100 **90%**

Write 1.45 as a percent

1.45×100 **145%**

Converting % to a Fraction

To write a percent as a fraction, drop the "%" and put the number over 100. Then, simplify the fraction if possible, using by eliminating shared prime factors or by dividing by the GCF

Write 60% as a fraction

$$\frac{60 \div 20}{100 \div 20} = \mathbf{\frac{3}{5}}$$

Write 8% as a fraction

$$\frac{8 \div 4}{100 \div 4} = \mathbf{\frac{2}{25}}$$

Write 160% as a fraction

$$\frac{160 \div 20}{100 \div 20} = \mathbf{\frac{8}{5}}$$

Write 100% as a fraction

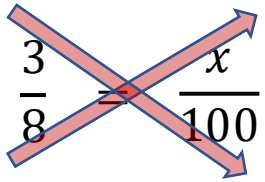
$$\frac{100 \div 100}{100 \div 100} = \mathbf{\frac{1}{1}}$$

Percent, Fractions, & Decimals

39%	$\frac{39}{100}$	0.39	*“thirty nine hundredths”*
79%	$\frac{79}{100}$	0.79	*“seventy nine hundredths”*
8%	$\frac{8}{100}$	0.08	*“eight hundredths”*

Using Proportions to find a %

Write $\frac{3}{8}$ as a percent



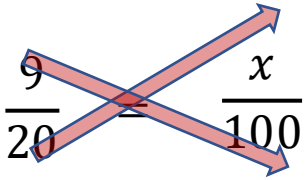
$$\frac{3}{8} = \frac{x}{100}$$

$$8 \cdot x = 3 \cdot 100$$

$$\frac{8x}{8} = \frac{300}{8}$$

$$x = \mathbf{37.5}\ \%$$

Write $\frac{9}{20}$ as a percent



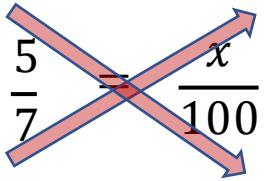
$$\frac{9}{20} = \frac{x}{100}$$

$$20 \cdot x = 9 \cdot 100$$

$$\frac{20x}{20} = \frac{900}{20}$$

$$x = \mathbf{45}\ \%$$

Write $\frac{5}{7}$ as a percent



$$\frac{5}{7} = \frac{x}{100}$$

$$\frac{7x}{7} = \frac{500}{7}$$

$$x = \mathbf{71.43}\ \%$$

You could also do this, to quickly find a %

Write $\frac{27}{39}$ as a percent

$$\frac{27}{39} = \frac{x}{100}$$

$$39 \cdot x = 27 \cdot 100$$

$$\frac{39x}{39} = \frac{2700}{39}$$

$$x = \mathbf{69.23}\ \%$$

Write $\frac{27}{39}$ as a percent

To quickly find the % of this fraction,

1) Convert the fraction to a decimal

2) Multiply this decimal by 100… *which means to move the decimal to the right 2 place values!*

$$\frac{27}{39} = 0.6923076923076923$$

$$= 69.23076923076923$$

$$= \mathbf{69.23}\%$$

Problem Solving and %

Percent Proportion

Amount: portion being compared to the entire whole
Base: Appears after the word “of”
%: Percentage (drop the “%” and put that number over 100)

For each percent problem-solving / word question, you will always be solving for ***one*** of either the **amount**, **base**, or **%**. (the 100 never changes as it is a constant within the proportion)

$$\frac{\text{amount}}{\text{base } (of)} = \frac{\%}{100}$$

Whatever the question is asking of you **IS** the unknown

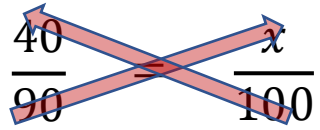
Problem Solving and %

40 is what percent of 90 ?

<u>Collect Evidence then solve for your unknown, x</u>

Amount	40
Base (of)	90
%	Unknown (x)

$$\frac{\text{amount}}{\text{base } (of)} = \frac{\%}{100}$$



$$90 \cdot x = 40 \cdot 100$$

$$\frac{90x}{90} = \frac{4000}{90}$$

$$x = \mathbf{44.4}\%$$

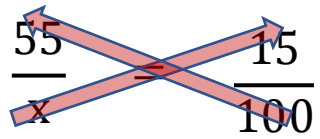
Problem Solving and %

15% of what number is 55 ?

$$\frac{\text{amount}}{\text{base } (of)} = \frac{\%}{100}$$

Collect Evidence then solve for your unknown, x

Amount	55
Base (of)	x
%	15



$$x \cdot 15 = 55 \cdot 100$$

$$\frac{15x}{15} = \frac{5500}{15}$$

$$x = \mathbf{366.6666}$$

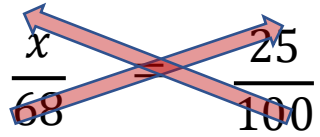
Problem Solving and %

What number is 25% of 68 ?

$$\frac{\text{amount}}{\text{base } (of)} = \frac{\%}{100}$$

Collect Evidence then solve for your unknown, x

Amount	x
Base (of)	68
%	25



$$x \cdot 100 = 68 \cdot 25$$

$$\frac{100x}{100} = \frac{1700}{100}$$

$$x = \mathbf{17}$$

Sales Tax

If the sales tax is set at 8.5%, what is the sales tax and the total amount due on a $62.80 purchase

Sales Tax $=$ Tax Rate $\times$ Purchase Price

$= \quad 0.085 \quad \times \quad 62.80$

$= \quad 5.338$

$= \quad \$\mathbf{5.34}$

TOTAL Price $=$ Purchase Price + Sales Tax

$= \quad 62.80 \quad + \quad \$5.34$

$= \quad \$\mathbf{68.14}$

$\therefore$With the sales tax rate set at 8.5%, the total sales tax amounts to $5.34 for a total price of $68.14

Sales Tax

The sales tax on a $304 pair of shoes is $12.16
Find the sales tax rate.

Recall: Sales Tax = Tax Rate × Purchase Price

<u>Collect Evidence then solve for your unknown</u>

Sales tax = $\$12.16$

Purchase Price = $\$304$

Tax Rate (expressed as a % later) = x

Sales Tax = Tax Rate × Purchase Price

$$12.16 = x \times 304$$

$$\frac{12.16}{304} = \frac{304x}{304}$$

$$0.04 = x$$

This is the tax rate shown as a decimal. To convert a decimal to a %, multiply by… 100

$$0.04 \times 100 = \mathbf{4}\%$$

∴The sales tax rate is set at 4% since the total sales tax is $12.16 on a $304 purchase of shoes

Commission

The People of Foundational Literacy: Michael Abraham

Michael recently got a job a local ski/snowboard shop as a salesperson. Part of his pay cheque is fully dependent on sales commission.

During his training days, **Michael** found out that his commission rate is set at 8.3 %

During his first week, **Michael** amazingly sold $3785.83 worth of merchandise, one of the highest amounts set by a brand new employee.

YOUR TASK: Find out the amount of commission **Michael** made in his first week of work

Commission

Michael's Commission Rate: 8.3%

Michael's Total Sales During First Week of Work: $3785.83

$$\text{Commission} = \text{commision rate} \times \text{total sales}$$

$$\frac{\%}{100}$$

$$= \quad 0.083 \times 3785.83$$

$$= \quad 314.22389$$

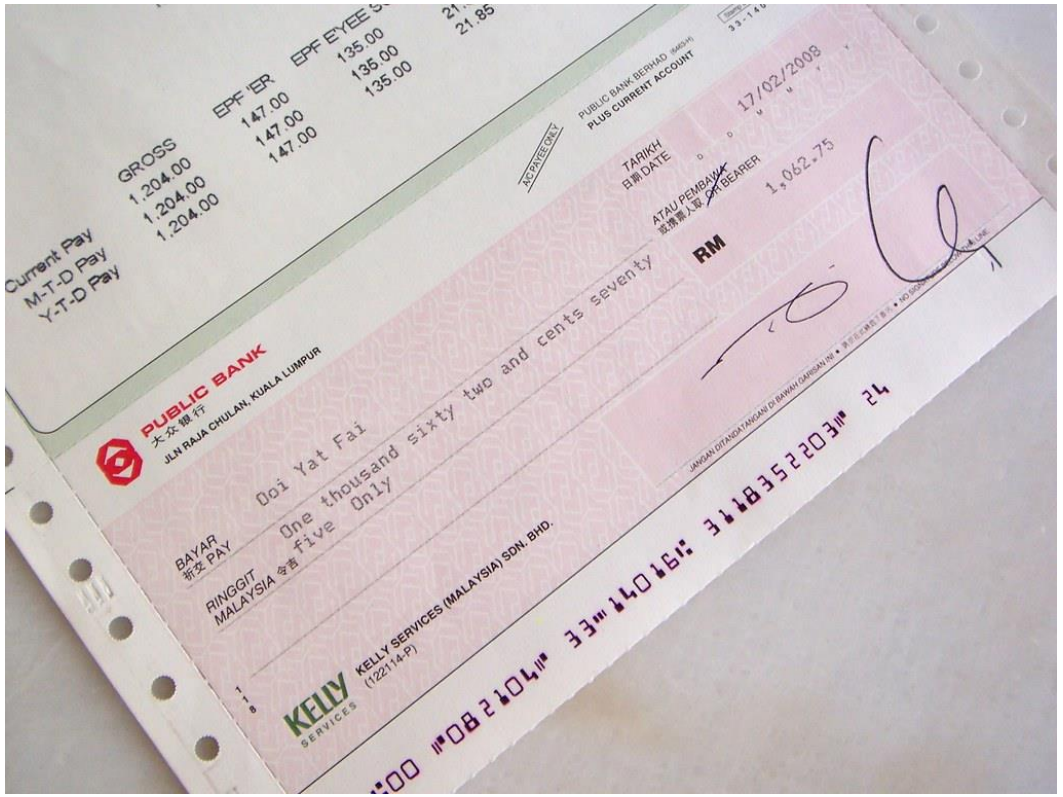
$$= \$\mathbf{314.22}$$

The People of Foundational Literacy: Michael Abraham

After a few months at his new job at the ski/snowboard shop, **Michael** has got into a groove of selling goods while providing the best knowledge to help customers meet their ski/snow shop.

During the most recent week, **Michael** is absolutely sure that his next pay cheque will be his largest commission yet, as he had sold $4568.23 , even more than the amount he sold on his first week.

He awakes to find that the amount deposited into his account was only $237.55 .



The People of Foundational Literacy: Michael Abraham

Michael's Total Commission: $\$237.55$

Michael's Total Sales: $\$4568.23$

Michael's Commission Rate: unknown (x)

$$\text{Commission} = \text{commision rate} \times \text{total sales}$$

$$237.55 = x \cdot 4568.23$$

$$\frac{237.55}{4568.23} = \frac{4568.23x}{4568.23}$$

$$0.052000446562454 2 = x$$

∴ Michael's employer's reduced his commission rate from 8.3% to

5.2%

Don't forget to multiply by 100…or move decimal to the right (2) times)

Discount & Sale Price

A new frying pan that normally sells for $40 is now on sale for 25% off. Calculate the amount of discount, and then calculate the new sale price.

$$\frac{x}{100}$$

Amount Discounted = Rate of Discount × Original Price

$$= 0.25 \times 40$$

$$= \mathbf{10}$$

∴ After a 25% discount is applied, the $40 frying pan will be discounted **$10**, for a sale price of **$30**

Sale Price = Original Price − Amount Discounted

$$= 40 - 10$$

$$= \mathbf{30}$$

Discount and Sale Price

An electric rice cooker that normally sells for $65 is now on sale for $48.75. Calculate discount rate.

Amount Discounted = Rate of Discount × Original Price

$$65 - 48.75 = x \times 65$$

$$\frac{16.25}{65} = \frac{65x}{65}$$

$$0.25 = x$$

$$0.25 = 25\,\%$$

Don't forget to multiply by 100!

∴The frying pan was 25% off